3.2.11 LRFD Slab Bridge Design

Slab bridges are defined as structures where the deck slab also serves as the main load-carrying component. The span-to-width ratios are such that these bridges may be designed for simple 1-way bending as opposed to 2-way plate bending. This design guide provides a basic procedural outline for the design of slab bridges using the LRFD Code and also includes a worked example.

The LRFD design process for slab bridges is similar to the LFD design process. Both codes require the main reinforcement to be designed for Strength, Fatigue, Control of Cracking, and Limits of Reinforcement. All reinforcement shall be fully developed at the point of necessity. The minimum slab depth guidelines specified in Table 2.5.2.6.3-1 need not be followed if the reinforcement meets these requirements.

For design, the Approximate Elastic or "Strip" Method for slab bridges found in Article 4.6.2.3 shall be used.

According to Article 9.7.1.4, edges of slabs shall either be strengthened or be supported by an edge beam which is integral with the slab. As depicted in Figure 3.2.11-1 of the Bridge Manual, the #5 d_1 bars which extend from the 34 in. F-Shape barrier into the slab qualify as shear reinforcement (strengthening) for the outside edges of slabs. When a 34 in. or 42 in. F-Shape barrier (with similar d_1 bars) is used on a slab bridge, its structural adequacy as an edge beam should typically only need to be verified. The barrier should not be considered structural. Edge beam design is required for bridges with open joints and possibly at stage construction lines. If the out-to-out width of a slab bridge exceeds 45 ft., an open longitudinal joint is required.

LRFD Slab Bridge Design Procedure, Equations, and Outline

Determine Live Load Distribution Factor (4.6.2.3)

Live Load distribution factors are calculated by first finding the equivalent width per lane that that will be affected. This equivalent width, or "strip width," in inches, is found using the following equations:

For single-lane loading or two lines of wheels (e.g. used for staged construction design considerations where a single lane of traffic is employed), the strip width E is taken as:

$$E = 10.0 + 5.0\sqrt{L_1 W_1} \qquad \text{(Eq. 4.6.2.3-1)}$$

For multiple-lane loading, the strip width E is taken as:

$$E = 84.0 + 1.44\sqrt{L_1 W_1} \le \frac{12.0W}{N_L} \qquad \text{(Eq. 4.6.2.3-2)}$$

When calculating E:

L_1 = modified span length, taken as the lesser of (a) the actual span length (ft.) or (b) 60 ft.

N_L = number of design lanes according to Article 3.6.1.1.1

W = actual edge-to-edge width of bridge (ft.)

W_1 = modified edge-to-edge width of bridge, taken as the lesser of (a) the actual edge to edge width W (ft.), or (b) 60 ft. for multiple-lane loading, 30 ft. for single-lane loading

According to Article 3.6.1.1.2, multiple presence factors shall not be employed when designing bridges utilizing Equations 4.6.2.3-1 and 4.6.2.3-2 as they are already embedded in the formulae.

The fatigue truck loading specified in Article 3.6.1.4 shall be distributed using the single-lane loaded strip width given in Equation 4.6.2.3-1, and the force effects shall be divided by 1.2 according to Article 3.6.1.1.2.

For slab bridges with skewed supports, the force effects may be reduced by a reduction factor r:

$r = 1.05 - 0.25\tan\theta \le 1.00$, where θ is the skew angle of the supports in degrees.

(Eq. 4.6.2.3-3)

The Department allows, but does not recommend, using the reduction factor for skewed bridges.

The live load distribution factor, with units "one lane, or two lines of wheels" per inch, is then taken as:

$$\text{LRFD DF (Single or Multiple Lanes Loaded)} = \frac{r}{E}$$

Or

$$\text{LRFD DF (Fatigue Truck Single Lane Loaded)} = \frac{r}{1.2E}$$

Note that the equations used to find the distribution factor in the AASHTO LRFD Code are in the units "one lane, or two lines for wheels" per inch, whereas the AASHTO LFD Code equations are in the units "lines of wheels" per inch. This is why the LRFD slab bridge live load distribution factor is 1/E (assuming r = 1.00), whereas the LFD slab bridge live load distribution factor is 1/2E.

These distribution factors apply to both shear and moment. Slab bridge slabs designed using the equivalent strip width method may be assumed to be adequate in shear (5.14.4.1), but edge beams on slab bridges require shear analysis.

Provisions for edge beam equivalent strip widths and load distribution are given in Article 4.6.2.1.4b.

Determine Maximum Factored Moments

In analyzing main reinforcement for slab bridges, three load combinations are used:

Strength I load combination is defined as:

$$M_{STRENGTH\ I} = \gamma_p(DC) + \gamma_p(DW) + 1.75(LL+IM+CE) \qquad \text{(Table 3.4.1-1)}$$

Where:

γ_p = For DC: maximum 1.25, minimum 0.90

For DW: maximum 1.50, minimum 0.65

Fatigue I load combination is defined as:

$M_{FATIGUE\ I}$ = 1.5(LL+IM+CE) (Table 3.4.1-1)

For the Fatigue I load combination, all moments are calculated using the fatigue truck specified in Article 3.6.1.4. The fatigue truck is similar to the HL-93 truck, but with a constant 30 ft. rear axle spacing. Impact or dynamic load allowance is taken as 15% of the fatigue truck load for this load combination (Table 3.6.2.1-1).

Fatigue II load combination is not checked for slab bridges.

Service I load combination is defined as:

$M_{SERVICE\ I}$ = 1.0(DC+DW+LL+IM+CE) (Table 3.4.1-1)

For these load combinations, loads are abbreviated as follows:

- CE = vehicular centrifugal force, including forces due to bridge deck superelevation
- DC = dead load of structural components (DC1) and non-structural attachments (DC2). This includes temporary concrete barriers used in stage construction. Parapets, curbs, and railings using the standard details found in Section 3.2.4 of the Bridge Manual need not be included in this value. Standard details for these components include additional longitudinal reinforcement and stirrups that, when built integrally with the slab, are adequate for self-support.
- DW = dead load of future wearing surface
- IM = impact or dynamic load allowance
- LL = vehicular live load

Design Reinforcement in Slab

Main reinforcement in slab bridges should be placed parallel to traffic except as allowed for some simple span skewed bridges. See Section 3.2.11 for the Bridge Manual for details. If possible, use the same size bars for all main reinforcement.

Four limit states are checked when designing main reinforcement: Flexural Resistance (5.7.3.2), Fatigue (5.5.3), Control of Cracking (5.7.3.4), and Limits of Reinforcement (5.7.3.3 & 5.5.4.2.1). These limit states should be checked at points of maximum stress and at theoretical cutoff points. See Figures 3.2.11-2 and 3.2.11-3 in the Bridge Manual for further guidance. As stated previously, shear analysis is unnecessary for designs using the distribution factors located in Article 4.6.2.3. The deformation control parameters of Article 2.5.2.6 may be used in determining of slab thickness in the TSL phase, but are not mandatory requirements for final design.

Distribution reinforcement is not designed, but rather is a percentage of the main reinforcement (5.14.4.1).

Check Flexural Resistance (5.7.3.2)

The factored resistance, M_r (k-in.), shall be taken as:

$$M_r = \phi M_n = \phi\left[A_s f_s\left(d_s - \frac{a}{2}\right)\right] \geq M_{STRENGTH\,1} \qquad \text{(Eqs. 5.7.3.2.1-1 \& 5.7.3.2.2-1)}$$

Where:

ϕ = Assumed to be 0.9, then checked in Limits of Reinforcement check

a = depth of equivalent stress block (in.), taken as $a = c\beta_1$

$c = \dfrac{A_s f_s}{0.85\beta_1 f'_c\, b}$ (in.) (Eqs. 5.7.3.1.1-4 or 5.7.3.1.2-4)

A_s = area of tension reinforcement in strip (in.^{2})

b = width of design strip (in.)

d_s = distance from extreme compression fiber to centroid of tensile reinforcement (in.)

f_s = stress in the mild steel tension reinforcement as specified at nominal flexural resistance (ksi). As specified in Article 5.7.2.1, if $c / d_s < 0.6$, then f_y may used in lieu of exact computation of f_s.

f'_c = specified compressive strength of concrete (ksi)

β_1 = stress block factor specified in Article 5.7.2.2

$$\therefore\ M_r = \phi M_n = \phi\left[A_s f_s\left(d_s - \frac{1}{2}\frac{A_s f_s}{0.85 f'_c\, b}\right)\right]$$

Check Control of Cracking (5.7.3.4)

The spacing of reinforcement, s (in.), in the layer closest to the tension face shall satisfy the following:

$$s \le \frac{700\gamma_e}{\beta_s f_{ss}} - 2d_c \quad \text{(Eq. 5.7.3.4-1)}$$

Where:

$$\beta_s = 1 + \frac{d_c}{0.7(h - d_c)}$$

d_c = thickness of concrete cover from extreme tension fiber to center of the flexural reinforcement located closest thereto (in.)

h = slab depth (in.)

f_{ss} = stress in mild steel tension reinforcement at service load condition

$$= \frac{M_{SERVICE\ I}}{A_s j d_s} \text{ (ksi)}$$

$$j = 1 - \frac{k}{3}$$

$$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n$$

ρ = $\dfrac{A_s}{bd_s}$

n = $\dfrac{E_s}{E_c}$, typically taken as 9 for 3.5 ksi concrete (C6.10.1.1.1b)

γ_e = 0.75 for Class 2 Exposure. C5.7.3.4 defines Class 2 Exposure as decks and any substructure units exposed to water.

Check Fatigue (5.5.3)

For fatigue considerations, concrete members shall satisfy:

$\gamma(\Delta f) \leq (\Delta F)_{TH}$

Where:

γ = load factor specified in Table 3.4.1-1 for the Fatigue I load combination
= 1.5

(Δf) = live load stress range due to fatigue truck (ksi)

$= \dfrac{\left| M^{+}_{FATIGUE\,I} - M^{-}_{FATIGUE\,I} \right|}{A_s jd_s}$

$(\Delta F)_{TH} = 24 - 0.33 f_{min}$ (Eq. 5.5.3.2-1)

Where:

f_{min} = algebraic minimum stress level, tension positive, compression negative (ksi). The minimum stress shall be taken as that from Service I factored dead loads (DC1 and DC2 with the inclusion of DW at the discretion of the designer), combined with that produced by $M^{-}_{FATIGUE\,I}$ in positive moment regions or $M^{+}_{FATIGUE\,I}$ in negative moment regions.

Check Limits of Reinforcement (5.7.3.3)

Check Maximum Reinforcement (5.7.3.3.1)

The 2006 Interims to the AASHTO LRFD Code do not explicitly state an absolute limit on the amount of reinforcement that can be used in a section. Rather, the code imposes reduced resistance factors for sections that experience very small amounts of strain i.e. are over-reinforced.

To determine whether or not a reduced resistance factor should be used, the tensile strain may be computed using the following equation:

$$\varepsilon_t = \frac{0.003(d_t - c)}{c} \qquad \text{(C5.7.2.1-1)}$$

Where:

d_t = distance from extreme compression fiber to centroid of bottom row of reinforcement (in.) As there is typically only one row of reinforcement in slab bridges, $d_t = d_s$.

$$c = \frac{A_s f_s}{0.85 \beta_1 f'_c\, b} \qquad \text{(Eq. 5.7.3.1.2-4)}$$

For $\varepsilon_t \geq 0.005$, the full value of $\phi = 0.9$ is used. (Fig. C5.5.4.2.1-1)

For $0.002 < \varepsilon_t < 0.005$, $\phi = 0.65 + 0.15\left(\frac{d_t}{c} - 1\right)$ (Eq. 5.5.4.2.1-2)

For $\varepsilon_t \leq 0.002$, $\phi = 0.75$ (Fig. C5.5.4.2.1-1)

The flexural resistance shall then be recalculated using this resistance factor, and a change in design made if necessary.

Check Minimum Reinforcement (5.7.3.3.2)

The minimum reinforcement shall be such that:

$M_r > 1.33M_{STRENGTH\ 1}$, or
$M_r > M_{cr}$

Where:

$M_{cr} = \gamma_3\gamma_1 S f_r$ (k-in.) (Eq. 5.7.3.3.2-1)

$S = \frac{1}{6}bh^2$ (in.^{3})

$f_r = 0.24\sqrt{f'_c}$ (ksi) (5.4.2.6)

γ_3 = 0.75 for A706, Grade 60 reinforcement

γ_1 = 1.6 for non-segmentally constructed bridges

Design Distribution Reinforcement (5.14.4.1)

Bottom distribution reinforcement is not designed, but rather is specified as a percentage of the main bottom reinforcement area. For slab bridges, the percentage is found using the following equations:

$$\frac{100}{\sqrt{L}} \leq 50\% \quad \text{(Eq. 5.14.4.1-1)}$$

Where L is the span length in feet.

Top distribution reinforcement is designed using the Temperature and Shrinkage requirements stated in Article 5.10.8. The required area of top distribution reinforcement, A_s, in square inches per foot width, shall be found using the following equation:

$$A_s \geq \frac{1.30bh}{2(b+h)f_y} \quad \text{(Eq. 5.10.8-1)}$$

$0.11 \leq A_s \leq 0.60$ (Eq. 5.10.8-2)

Where h is the slab depth (in.), and b is the total width of the slab (in).

While Temperature and Shrinkage reinforcement is not necessary for members with a depth of less than 18 inches (5.10.8), it is IDOT policy that it shall be used.

Spacing for reinforcement designed for Shrinkage and Temperature shall not exceed 18 inches center-to-center, nor shall it exceed three times the slab thickness.

Development of Reinforcement (5.11)

Provisions for development of reinforcement are found in Article 5.11. See also Figures 3.2.11-2 and 3.2.11-3 of the Bridge Manual for additional guidance on development lengths, detailing and bar cutoffs.

Edge Beams (9.7.1.4)

Edges of slabs at parapets (as described above) should be verified as adequate and at open joints they should be fully designed. Edge beams shall be checked or designed for both shear and moment.

The width of equivalent strips and distribution of loads for edge beams shall be determined from Article 4.6.2.1.4b.

The moment design for an edge beam is similar to that for the slab described above. Shear design shall be as follows:

<u>Determine Shear Resistance</u> (5.8.3.3)

The factored concrete shear resistance, ϕV_c (kips) (ϕ = 0.9), shall be found using the following equation:

$$\phi V_c = \phi 0.0316\beta\sqrt{f'_c}\, b_v d_v \qquad \text{(Eq. 5.8.3.3-3)}$$

Where:

b_v = effective web width (in.)

d_v = effective shear depth, taken as the greater of $0.9d_s$ or $0.72h$ (in.)

β = may be conservatively taken as 2.0 for slab bridges, regardless of slab depth.

At edges of slabs with F-Shape parapets and standard IDOT reinforcement, the factored shear steel resistance, ϕV_s (ϕ = 0.9), shall be found using the following equation:

$$\phi V_s = \phi \frac{A_v f_y d_v}{s} \qquad \text{(Eq. 5.8.3.3-4 – Simplified per 5.8.3.4.1)}$$

Where:

s = spacing of stirrups (in)

A_v = area of shear reinforcement within a distance s (in.^{2})

d_v = effective shear depth, taken as the greater of $0.9d_s$ or $0.72h$ (in.)

f_y = yield strength of steel (ksi)

When necessary, an edge beam may be thickened at an open joint using a concrete haunch until shear capacity is met.

LRFD Slab Bridge Design Example: Two-Span Slab Bridge, No Skew

Design Stresses

f'_c = 3.5 ksi

f_y = 60 ksi

w_c = weight of reinforced concrete = 0.150 kcf

E_c = $33000 K_1 w_c^{1.5} \sqrt{f'_c}$ (Eq. 5.4.2.4-1)

Where K_1 is a correction factor for source of aggregate, assumed to be 1.0 unless proven otherwise. In this example, K_1 = 1.0.

$$\therefore \quad E_c = 33000(1.0)(0.150\ \text{kcf})^{1.5}\sqrt{3.5\ \text{ksi}} = 3587\ \text{ksi}$$

Bridge Data

Span Lengths:	Two 36 ft. Spans
Bridge Slab Width:	32 ft. Out-To-Out Including F-Shape Parapets
Slab Thickness:	16 in.
Future Wearing Surface:	50 psf
Skew:	No Skew

Note: Design at theoretical cutoff points not included in this example.

Determine Live Load Distribution Factors (4.6.2.3)

<u>For Multiple-Lanes Loaded (Used for Strength I Design)</u>

$$E = 84.0 + 1.44\sqrt{L_1 W_1} \le \frac{12.0W}{N_L} \qquad \text{(Eq. 4.6.2.3-2)}$$

Where:

N_L = 2 lanes

W = 32 ft.

L_1 = 36 ft.

W_1 = 32 ft.

$$84.0 + 1.44\sqrt{L_1 W_1} = 84.0 + 1.44\sqrt{(36\ \text{ft.})(32\ \text{ft.})} = 132.88\ \frac{\text{in.}}{\text{lane}}$$

$$\frac{12.0W}{N_L} = \frac{12.0(32)}{2} = 192.00\ \frac{\text{in.}}{\text{lane}}$$

$$\therefore\ E = 132.88\ \frac{\text{in.}}{\text{lane}}$$

$$\text{Live Load Distribution Factor} = \frac{1\,\text{lane}}{132.88\text{ in.}}\left(\frac{12\text{ in.}}{\text{ft.}}\right) = 0.090\,\frac{\text{lanes}}{\text{ft. of slab width}}$$

For Single-Lane Loaded (Modified for Fatigue Truck)

$$E \quad = \quad 10.0 + 5.0\sqrt{L_1W_1} \qquad \text{(Eq. 4.6.2.3-1)}$$

Where:

L_1 = 36 ft.

W_1 = 30 ft.

$$E = 10.0 + 5.0\sqrt{(36\text{ ft.})(30\text{ ft.})} = 174.32\ \frac{\text{in.}}{\text{lane}}$$

$$\text{Live Load Distribution Factor (Non-Fatigue)} = \frac{1\,\text{lane}}{174.32\text{ in.}}\left(\frac{12\text{ in.}}{\text{ft.}}\right) = 0.069\,\frac{\text{lanes}}{\text{ft. of slab width}}$$

$$\text{Live Load Distribution Factor (Modified for Fatigue)} = \frac{0.069}{1.2} = 0.058\ \frac{\text{lanes}}{\text{ft. of slab width}}$$

Note: Edge beam load distribution considered at the end of example.

Determine Maximum Factored Moments

Span 2 factored loads are symmetric to span 1.

Using the full bridge width live load distribution factors, the following moments have been calculated (k-ft.) for a one-foot-wide strip width:

Pt.	$M^{+}_{STRENGTH\ I}$	$M^{-}_{STRENGTH\ I}$	$M^{+}_{SERVICE\ I}$	$M^{-}_{SERVICE\ I}$	$M^{+}_{FATIGUE\ I}$	$M^{-}_{FATIGUE\ I}$
0.4	110.6	0.4	68.9	11.9	25.4	-4.8
1.0	-34.4	-118.4	-40.5	-78.1	0.0	-23.6

Design Positive Moment Reinforcement

Check Flexural Resistance @ 0.4 Span 1 (5.7.3.2)

$$M_r = \phi M_n = \phi\left[A_s f_s\left(d_s - \frac{a}{2}\right)\right] \geq M_{STRENGTH\,I}$$ (Eqs. 5.7.3.2.1-1 & 5.7.3.2.2-1)

Assume #9 bars, solve for A_s:

b = 12 in.

d_s = 16 in. – 1 in. clear – 0.5(1.128 in. bar diameter) = 14.44 in.

f_s = Assume 60 ksi, if $c / d_s < 0.6$ then assumption is valid (5.7.2.1)

f'_c = 3.5 ksi

ϕ = Assumed to be 0.9, then checked in Limits of Reinforcement check

β_1 = 0.85 (5.7.2.2)

$$c = \frac{A_s(60\text{ ksi})}{0.85(0.85)(3.5\text{ ksi})(12\text{ in.})} = 1.98A_s\text{ in.}$$ (Eqs. 5.7.3.1.1-4 or 5.7.3.1.2-4)

$a = c\beta_1 = 0.85(1.98A_s) = 1.68A_s$ in.

$$M_r = M^{+}_{STRENGTH\,I} = 110.6\text{ k-ft.}\left(\frac{12\text{ in.}}{\text{ft.}}\right) = 1327.2\text{ k-in.}$$

$$1327.2\text{ k-in.} = (0.9)\left[A_s(60\text{ ksi})\left(14.44\text{ in.} - \frac{1.68A_s\text{ in.}}{2}\right)\right]$$

Solving for A_s gives A_s = 1.92 in.^{2} Try #9 bars @ 6 in. center-to-center spacing, A_s = 2.00 in.^{2}

c = 1.98 A_s = 1.98(2.00 in.^{2}) = 3.96 in.

d_s = 14.44 in.

$$\frac{c}{d_s} = \frac{3.96\text{ in.}}{14.44\text{ in.}} = 0.27 < 0.6 \quad \therefore \text{Assumption of } f_s = f_y = 60\text{ ksi is valid.}$$

Check Control of Cracking @ 0.4 Span 1 (5.7.3.4)

$$s \leq \frac{700\gamma_e}{\beta_s f_{ss}} - 2d_c$$ (Eq. 5.7.3.4-1)

Where:

d_c = 1 in. clear + 0.5(1.128 in. bar diameter) = 1.564 in.

h = 16 in.

$$\beta_s = 1 + \frac{1.564 \text{ in.}}{0.7(16 \text{ in.} - 1.564 \text{ in.})} = 1.155$$

$$\rho = \frac{2.00 \text{ in.}^2}{(12 \text{ in.})(14.44 \text{ in.})} = 0.0115$$

n = 9

$$k = \sqrt{[(0.0115)(9)]^2 + 2(0.0115)(9)} - (0.0115)(9) = 0.363$$

$$j = 1 - \frac{0.363}{3} = 0.879$$

$$f_{ss} = \frac{(68.9 \text{ k} - \text{ft.})\left(\frac{12 \text{ in.}}{\text{ft.}}\right)}{(2.00 \text{ in.}^2)(0.879)(14.44 \text{ in.})} = 32.57 \text{ ksi}$$

γ_e = 0.75

$$\frac{700\gamma_e}{\beta_s f_s} - 2d_c = \frac{700(0.75)}{(1.155)(32.57)} - 2(1.564) = 10.83 \text{ in.}$$

s = 6 in. < 10.83 in. O.K.

$\therefore$ #9 bars @ 6 in. center-to-center spacing is adequate to control cracking.

Check Fatigue @ 0.4 Span 1 (5.5.3)

$\gamma(\Delta f) \leq (\Delta F)_{TH}$ (5.5.3.1-1)

Where:

$$\gamma(\Delta f) = \frac{\left|M^{+}_{\text{FATIGUE I}} - M^{-}_{\text{FATIGUE I}}\right|}{A_s j d_s}$$

Note that $M^{+}_{\text{FATIGUE I}}$ and $M^{-}_{\text{FATIGUE I}}$ already include a 1.5 factor for γ.

$$M^{+}_{FATIGUE\ I} = 25.4\text{ k-ft.}\left(\frac{12\text{ in.}}{\text{ft.}}\right) = 304.8\text{ k-in.}$$

$$M^{-}_{FATIGUE\ I} = -4.8\text{ k-ft.}\left(\frac{12\text{ in.}}{\text{ft.}}\right) = -57.6\text{ k-in.}$$

$$\gamma(\Delta f) = \frac{\left|(304.8\text{ k}-\text{in.}) - (-57.6\text{ k}-\text{in.})\right|}{(2.00\text{ in.}^2)(0.879)(14.44\text{ in.})} = 14.27\text{ ksi}$$

$$(\Delta F)_{TH} = 24 - 0.33 f_{min}$$ (Eq. 5.5.3.2-1)

From the force envelope Service I dead load moment is 22.6 k-ft.

$$f_{min} = \frac{(-4.8\text{ k}-\text{ft.} + 22.6\text{ k}-\text{ft.})\left(\frac{12\text{ in.}}{\text{ft.}}\right)}{(2.00\text{ in.}^2)(0.879)(14.44\text{ in.})} = 8.41\text{ ksi}$$

$$24 - 0.33 f_{min} = 24 - 0.33(8.41) = 21.22\text{ ksi}$$

14.27 ksi < 21.22 ksi O.K.

∴ #9 bars @ 6 in. center-to-center spacing is adequate for fatigue limit state.

Check Limits of Reinforcement (5.7.3.3)

Check Maximum Reinforcement (5.7.3.3.1)

$$\varepsilon_t = \frac{0.003(d_t - c)}{c}$$ (C5.7.2.1-1)

Where:

c = 3.96 in.

d_t = d_s = 14.44 in.

$$\varepsilon_t = \frac{0.003(14.44\text{ in.} - 3.96\text{ in.})}{3.96\text{ in.}} = 0.008$$

0.008 > 0.005, $\therefore$ no reduction in resistance factors is required and Ultimate Moment Capacity computations are valid.

Check Minimum Reinforcement (5.7.3.3.2)

The minimum reinforcement shall be such that:

$M_r > M_{cr}$

Where:

$M_{cr} = \gamma_3\gamma_1 S f_r$ (k-in.) (Eq. 5.7.3.3.2-1)

γ_3 = 0.75 for A706, Grade 60 reinforcement

γ_1 = 1.6 for non-segmentally constructed bridges

$S = \frac{1}{6}(12\text{ in.})(16\text{ in.})^2 = 512\text{ in.}^3$

$f_r = 0.24\sqrt{3.5\text{ ksi}} = 0.449\text{ ksi}$ (5.4.2.6)

$M_{cr} = 0.75(1.6)(512\text{ in.}^3)(0.449\text{ ksi}) = 275.9\text{ k-in.}$

$$M_r = 0.9\left(2.00\text{ in.}^2\right)(60\text{ ksi})\left[14.44\text{ in.} - \frac{\left(2.00\text{ in.}^2\right)(60\text{ ksi})}{2(0.85)(12\text{ in.})(3.5\text{ ksi})}\right] = 1378.0\text{ k-in.}$$

1378.0 k-in. > 275.9 k-in. O.K.

Design Bottom Distribution Reinforcement (5.14.4.1)

Bottom distribution reinforcement is a percentage of the main bottom reinforcement:

$$\frac{100}{\sqrt{L}} \leq 50\% \qquad \text{(Eq. 5.14.4.1-1)}$$

Where L = 36 ft.

$$\frac{100}{\sqrt{36}} = 16.67\%$$

A_s = 0.1667(2.00 in.^{2}) = 0.333 in.^{2}

Use #5 bars @ 11 in. center-to-center spacing, A_s = 0.338 in.^{2}

Design Negative Moment Reinforcement

Check Flexural Resistance @ 1.0 Span 1 (5.7.3.2)

$$M_r = \phi M_n = \phi\left[A_s f_s\left(d_s - \frac{a}{2}\right)\right] \geq M_{STRENGTH\,1}$$ (Eqs. 5.7.3.2.1-1 & 5.7.3.2.2-1)

Assume #9 bars, solve for A_s:

b = 12 in.

d_s = 16 in. – (2.25 + 0.25) in. clear – 0.5(1.128 in. bar diameter) = 12.94 in.

f_s = Assume 60 ksi, if $c / d_s < 0.6$ then assumption is valid (5.7.2.1)

f'_c = 3.5 ksi

ϕ = Assumed to be 0.9, then checked in Limits of Reinforcement check

$$c = \frac{A_s(60\text{ ksi})}{0.85(0.85)(3.5\text{ ksi})(12\text{ in.})} = 1.98A_s$$ (Eq. 5.7.3.1.1-4 or 5.7.3.1.2-4)

$a = c\beta_1 = 0.85(1.98A_s) = 1.68A_s$

$$M_r = M^-_{STRENGTH\,I} = 118.4\text{ k-ft.}\left(\frac{12\text{ in.}}{\text{ft.}}\right) = 1420.8\text{ k-in.}$$

$$1420.8\text{ k-in.} = (0.9)\left[A_s(60\text{ ksi})\left(12.94\text{ in.} - \frac{1.68A_s\text{ in.}}{2}\right)\right]$$

Solving for A_s gives A_s = 2.41 in.^{2} Try #9 bars @ 4.5 in. center-to-center spacing. A_s = 2.67 in.^{2}

c = 1.98 A_s = 1.98(2.67 in.^{2}) = 5.29 in.

d_s = 12.94 in.

$$\frac{c}{d_s} = \frac{5.29\text{ in.}}{12.94\text{ in.}} = 0.41 < 0.6 \quad \therefore \text{Assumption of } f_s = f_y = 60\text{ ksi is valid.}$$

Check Control of Cracking @ 1.0 Span 1 (5.7.3.4)

$$s \le \frac{700\gamma_e}{\beta_s f_{ss}} - 2d_c \qquad \text{(Eq. 5.7.3.4-1)}$$

Where:

d_c = (2.25 + 0.25) in. clear + 0.5(1.128 in. bar diameter) = 3.064 in.

h = 16 in.

$$\beta_s = 1 + \frac{3.064 \text{ in.}}{0.7(16 \text{ in.} - 3.064 \text{ in.})} = 1.338$$

$$\rho = \frac{2.67 \text{ in.}^2}{(12 \text{ in.})(12.94 \text{ in.})} = 0.0172$$

n = 9

$$k = \sqrt{[(0.0172)(9)]^2 + 2(0.0172)(9)} - (0.0172)(9) = 0.423$$

$$j = 1 - \frac{0.423}{3} = 0.859$$

$$f_{ss} = \frac{(78.1 \text{ k} - \text{ft.})\left(\frac{12 \text{ in.}}{\text{ft.}}\right)}{(2.67 \text{ in.}^2)(0.859)(12.94 \text{ in.})} = 31.6 \text{ ksi}$$

γ_e = 0.75

$$\frac{700\gamma_e}{\beta_s f_s} - 2d_c = \frac{700(0.75)}{(1.338)(31.6)} - 2(3.064) = 6.29 \text{ in.}$$

s = 4.5 in. < 6.29 in. O.K.

$\therefore$#9 bars @ 4.5 in. center-to-center spacing is adequate to control cracking.

Check Fatigue @ 1.0 Span 1 (5.5.3)

$$\gamma(\Delta f) \le (\Delta F)_{TH} \qquad (5.5.3.1\text{-}1)$$

Where:

$$\gamma(\Delta f) = \frac{\left|M^{+}_{FATIGUE\ I} - M^{-}_{FATIGUE\ I}\right|}{A_s j d_s}$$

Note that $M^{+}_{FATIGUE\ I}$ and $M^{-}_{FATIGUE\ I}$ already include a 1.5 factor for γ.

$$M^{+}_{FATIGUE\ I} = 0 \text{ k-ft.}\left(\frac{12 \text{ in.}}{\text{ft.}}\right) = 0 \text{ k-in.}$$

$$M^{-}_{FATIGUE\ I} = -23.6 \text{ k-ft.}\left(\frac{12 \text{ in.}}{\text{ft.}}\right) = -283.2 \text{ k-in.}$$

$$\gamma(\Delta f) = \frac{\left|(0 \text{ k}-\text{in.}) - (-283.2 \text{ k}-\text{in.})\right|}{(2.67 \text{ in.}^2)(0.859)(12.94 \text{ in.})} = 9.54 \text{ ksi}$$

$$(\Delta F)_{TH} = 24 - 0.33 f_{min} \qquad \text{(Eq. 5.5.3.2-1)}$$

From the force envelope Service I dead load moment is -40.5 k-ft.

$$f_{min} = \frac{(0 \text{ k}-\text{ft.} + 40.5 \text{ k}-\text{ft.})\left(\frac{12 \text{ in.}}{\text{ft.}}\right)}{(2.67 \text{ in.}^2)(0.859)(12.94 \text{ in.})} = 16.38 \text{ ksi}$$

$$24 - 0.33 f_{min} = 24 - 0.33(16.38) = 18.59 \text{ ksi}$$

9.54 ksi < 18.59 ksi O.K.

$\therefore$ #9 bars @ 4.5 in. center-to-center spacing is adequate for fatigue limit state.

Check Limits of Reinforcement (5.7.3.3)

Check Maximum Reinforcement (5.7.3.3.1)

$$\varepsilon_t = \frac{0.003(d_t - c)}{c} \qquad \text{(C5.7.2.1-1)}$$

Where:

c = 5.29 in.

d_t = d_s = 12.94 in.

$$\varepsilon_t = \frac{0.003(12.94\text{ in.} - 5.29\text{ in.})}{5.29\text{ in.}} = 0.004$$

$$0.004 < 0.005,\ \therefore \phi = 0.65 + 0.15\left(\frac{d_t}{c} - 1\right)$$

$$0.65 + 0.15\left(\frac{d_t}{c} - 1\right) = 0.65 + 0.15\left(\frac{12.94}{5.29} - 1\right) = 0.87$$

Re-evaluate Ultimate Strength of section

$$M_r = \phi M_n = \phi\left[A_s f_y\left(d_s - \frac{a}{2}\right)\right] \geq M_{STRENGTH\,1}$$

$$\phi M_n = 0.87(2.67\text{ in.}^2)(60\text{ ksi})\left[12.94\text{ in.} - \frac{(2.67\text{ in.}^2)(60\text{ ksi})}{2(0.85)(12\text{ in.})(3.5\text{ ksi})}\right] = 1490.8\text{ k-in.}$$

$$M^{-}_{STRENGTH\,I} = 118.4\text{ k-ft.}\left(\frac{12\text{ in.}}{\text{ft.}}\right) = 1420.8\text{ k-in.} < 1490.8\text{ k-in. OK}$$

The Ultimate Strength of the section is still adequate using a reduced ϕ factor.

Check Minimum Reinforcement (5.7.3.3.2)

The minimum reinforcement shall be such that:

$M_r > M_{cr}$

Where:

$M_{cr} = \gamma_3\gamma_1 S f_r$ (k-in.) (Eq. 5.7.3.3.2-1)

γ_3 = 0.75 for A706, Grade 60 reinforcement

γ_1 = 1.6 for non-segmentally constructed bridges

$S = \frac{1}{6}(12\text{ in.})(16\text{ in.})^2 = 512\text{ in.}^3$

$f_r = 0.24\sqrt{3.5\text{ ksi}} = 0.449\text{ ksi}$ (5.4.2.6)

M_{cr} = 0.75(1.6)(512 in.^{3})(0.449 ksi) = 275.9 k-in.

M_r = 1490.8 k-in. (see Maximum Reinforcement check)

1490.8 k-in. > 275.9 k-in. O.K.

Design Top Distribution Reinforcement (5.10.8)

Top distribution reinforcement is designed using the Temperature and Shrinkage requirements stated in Article 5.10.8:

$$A_s \geq \frac{1.30bh}{2(b+h)f_y} \qquad \text{(Eq. 5.10.8-1)}$$

Where:

b = 12 in/ft. $\times$ 32 ft. = 384 in.

h = 16 in.

f_y = 60 ksi

$$A_s \geq \frac{1.30(384\text{ in.})(16\text{ in.})}{2(384\text{ in.}+16\text{ in.})(60\text{ ksi})} = 0.166\text{ in.}^2\text{/ft.}$$

Use #5 bars @ 18 in. maximum allowable center-to-center spacing (5.10.8), A_s = 0.21 in.^{2}

Verify Edge Beam (Parapet) Adequacy

<u>Determine Distribution Factor</u> (4.6.2.1.4b)

The design strip width, E, for 1-line of wheel loading (½ lane) shall be taken as:

Width of parapet base + 12 in. + ¼ of strip width for a single lane loaded, not to exceed ½ the full strip width or 72 in.

E = 19 in + 12 in. + ¼(174.32) = 74.58 in. > 72 in.

$\therefore$ E = 72 in.

$$\text{Live Load Distribution Factor} = \frac{\tfrac{1}{2}\text{ lane}}{72\text{ in.}}\left(\frac{12\text{ in.}}{\text{ft.}}\right) = 0.083\,\frac{\text{lanes}}{\text{ft. of slab width}}$$

Since this Distribution Factor is less than that used above for design of the interior slab and the effective depth of the edge beam is equal to that of the slab, the edge beam is OK for ultimate moment design by inspection. It is also adequate for fatigue by inspection.

Determine Shear Resistance (5.8.3.3)

Conservatively, the design shear over the pier is:

$V_{STRENGTH\ I}$ = 20.5 kips

The concrete shear resistance, ϕV_c, shall be found using the following equation:

$$\phi V_c = \phi 0.0316\beta\sqrt{f'_c}\, b_v d_v$$ (Eq. 5.8.3.3-3)

Where:

ϕ = 0.9 for shear (5.5.4.2)

b_v = 12 in. strip width

d_v can be taken as:

0.72h = 0.72(16 in.) = 11.52 in.

β = 2.0

$$\phi V_c = 0.9 \times 0.0316(2.0)\sqrt{3.5\ \text{ksi}}(12\ \text{in.})(11.52\ \text{in.}) = 14.71\ \text{kips}$$

The shear steel resistance, ϕV_s, shall be found using the following equation:

$$\phi V_s = \phi \frac{A_v f_y d_v}{s}$$ (Eq. 5.8.3.3-4 – Simplified per 5.8.3.4.1)

Where:

ϕ = 0.9 for shear (5.5.4.2)

s = 11 in.

A_v = 0.31 in.^{2} (taking only one stirrup as effective in 12 in.)

d_v = 11.52 in.

f_y = 60 ksi

$$\phi V_s = \frac{0.9 \times 0.31 in^2 \times 60\, ksi \times 11.52\, in}{11 in} = 17.53$$

14.71 + 17.53 kips = 32.24 kips > 20.5 kips O.K.