3.15 Seismic Design

This design guide focuses on simple and practical techniques which can be used for the analysis and design of typical bridges in Illinois for earthquake loadings. More sophisticated methods are not discouraged by the Department given that the designer has the expertise. The primary format of this guide is to provide examples with discussion and commentary. Example Bridges No. 1 and No. 4 are the most complete. General guidance is also provided beyond the scope of the specific example structures for many other bridge types and potential design scenarios. The intent is to cover as broad a range of subjects as possible in this relatively short forum. Examples 1, 2 & 3 focus on bridges commonly built on the State and Local Systems. Examples 2 and 3 either illustrate a variation on Example 1 which requires a demonstration of a separate set of methods and calculations for clarity, or builds upon concepts already presented. Example 4 deals with a class of bridges historically constructed on the Local Bridge System, single and multi-span simply supported PPC deck beam structures. These bridges have some special characteristics and design considerations (which includes the flexible design option described in Section 3.15.8 of the Bridge Manual) that are unique compared to other structure types built in the State. Taken as a whole, this design guide is intended as an abbreviated primer on the design of typical bridges in Illinois for earthquake loadings.

The design of superstructure-to-substructure connections along with seat widths (Level 1 and Level 2 Redundancies in the Department's ERS framework) is straightforward and not covered in Examples 1 to 3. However, Example 4 does because simply supported PPC deck beam bridges have some special design considerations for the Level 1 and 2 Redundancies. See also Sections 3.7 and 3.15 of the Bridge Manual for more information.

This design guide deals with both the 1000 yr. (LRFD) and 500 yr. (LFD) design return period earthquakes. Both will still be relevant for bridges in Illinois for the foreseeable future with the importance of the 500 yr. event decreasing over time. Example 1 juxtaposes the seismic design methods and calculations for an identical bridge for both the 1000 yr. and 500 yr. events in order to highlight the differences and similarities between the two, and serves as a transitional reference for the designer. Example 1 also demonstrates that the increases in concrete member strengths, the number of piles required, etc. when going from the 500 yr. to the 1000 yr. design event are not overly dramatic.

The following is an outline of provided examples:

Example 1

3-Span Wide Flange Bridge with Pile Supported Multiple Circular Column Bents, Open Pile Supported Stub Abutments (Pile Bents), and No Skew

1. *Determination of Bridge Period – Transverse Direction*
    a. Weight of Bridge for Seismic Calculations
    b. Global Transverse Structural Model of the Bridge
    c. Transverse Pier Stiffness for Un-cracked and Cracked Columns
    d. Transverse Abutment Stiffness
    e. Transverse Superstructure Stiffness
    f. Uniform Load Method Transverse Period Determination for Un-cracked Columns
    g. Uniform Load Method Transverse Period Determination for Cracked Columns
2. *Determination of Bridge Period – Longitudinal Direction*
    a. Weight and Global Longitudinal Structural Model of the Bridge
    b. Longitudinal Pier Stiffness for Un-cracked and Cracked Columns
    c. Uniform Load Method Longitudinal Period Determination for Un-cracked and Cracked Columns
3. *Determination of Base Shears – 500 Year Design Earthquake Return Period*
    a. Design Response Spectrum (LFD)
    b. Transverse Base Shear
    c. Longitudinal Base Shear
4. *Determination of Base Shears – 1000 Year Design Earthquake Return Period*
    a. Design Response Spectrum (LRFD)
    b. Transverse Base Shear
    c. Longitudinal Base Shear
5. *Frame Analysis and Columnar Seismic Forces for Multiple Column Bent – 500 and 1000 Year Design Earthquake Return Period*
    a. Pier Forces – Dead Load
    b. Pier Forces – Transverse Overturning
    c. Pier Forces – Transverse Frame Action
    d. Pier Forces – Longitudinal Cantilever

6. *Seismic Design Forces for Multiple Column Bent Including R-Factor, P-Δ, and Combination of Orthogonal Forces – 500 and 1000 Year Design Earthquake Return Period*
    a. R-Factor
    b. P-Δ
    c. Summary and Combination of Orthogonal Column Forces Used for Design
7. *Column Design Including Overstrength Plastic Moment Capacity – 500 and 1000 Year Design Earthquake Return Period*
    a. Column Design for Axial Force and Moment
    b. Column Design for Shear
    c. 1000 Year Return Period Plastic Shear Determination Using Overstrength
8. *Pile Design Overview*

Example 2

Example 1 Bridge with a Skew of 30° for the 500 Year Design Earthquake Return Period

1. *Determination of Bridge Periods and Base Shears – 500 Year Design Earthquake Return Period*
2. *Frame Analysis and Columnar Seismic Forces for Multiple Column Bent – 500 Year Design Earthquake Return Period*
    a. Pier Forces – Dead Load
    b. Pier Forces from Global Transverse Base Shear
    c. Pier Forces from Global Longitudinal Base Shear
3. *Seismic Design Forces for Multiple Column Bent Including R-Factor, P-Δ, and Combination of Orthogonal Forces – 500 Year Design Earthquake Return Period*
    a. R-Factor
    b. P-Δ
    c. Summary and Combination of Orthogonal Column Forces Used for Design

Example 3

Overview of Bents with Rectangular or Trapezoidal Columns

1. *Overview of Seismic Design of Multiple Column Bents with Rectangular or Trapezoidal Columns for Bridges with No Skew*
2. *Overview of Seismic Design of Multiple Column Bents with Rectangular or Trapezoidal Columns for Bridges with Skew*

Example 4

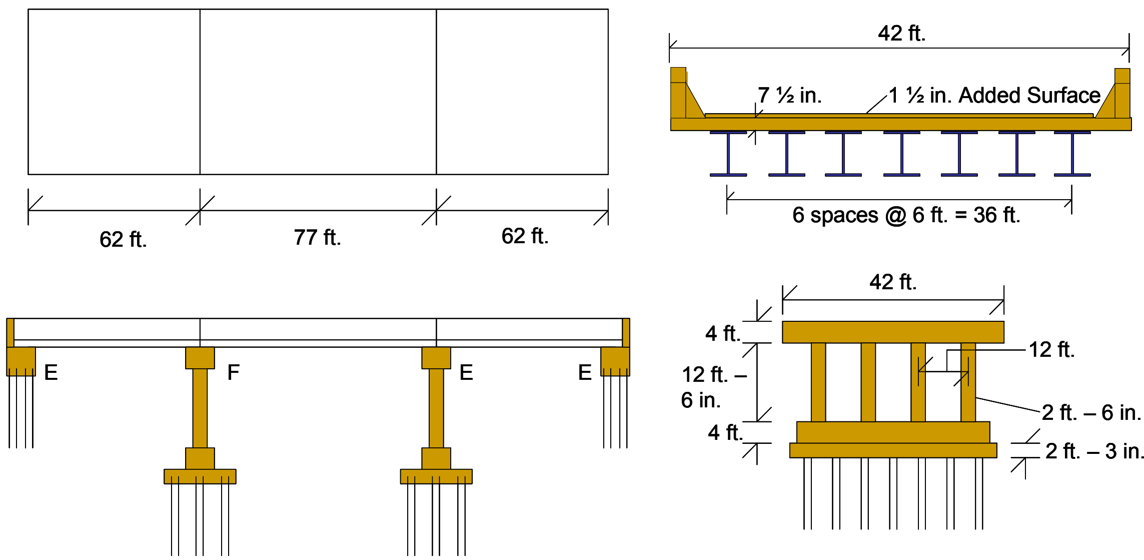
Design of a Simply Supported Multi-Span PPC Deck Beam Bridge for 1000 yr. Design Return Period Earthquake Using the Flexible Option

5. *Frame Analysis and Pile (Columnar) Seismic Forces for Abutments*
    a. Abutment Forces – Dead Load
    b. Abutment Forces – Transverse Overturning
    c. Abutment Forces – Transverse Frame Action
    d. Abutment Forces – Longitudinal Cantilever
6. *Seismic Design Forces for Pile Bent Including R-Factor, P-Δ, and Combination of Orthogonal Forces*
    a. R-Factor
    b. P-Δ
    c. Summary and Combination of Orthogonal Column Forces Used for Design
7. *Seismic Design Forces for Abutment Including R-Factor, P-Δ, and Combination of Orthogonal Forces*
    a. R-Factor
    b. P-Δ
    c. Summary and Combination of Orthogonal Column Forces Used for Design
8. *Combined Axial Force and Bi-Axial Bending Structural Capacity Check for Piles in Bents*
    a. Load Case 1 – Longitudinal Dominant
    b. Load Case 2 – Transverse Dominant
9. *Combined Axial Force and Bi-Axial Bending Structural Capacity Check for Piles in Abutments*
    a. Load Case 1 – Longitudinal Dominant
    b. Load Case 2 – Transverse Dominant
10. *Discussion of Flexible Versus Standard Design Options*
11. *Pile Shear Structural Capacity Check, and Pile Connection Details, and Cap Reinforcement Details*
    a. Shear Capacity Check of HP Piles
    b. Anchorage Details at Piers and Abutments for HP Piles
    c. Added Pier Cap Confinement Reinforcement
12. *Dowel Bar Connection of Beams to Pier and Abutment Caps*
13. *Minimum Support Length (Seat Width) Requirements at Piers and Abutments*
14. *Overview of Example Bridge Design With Metal Shell Piles*

a. Determination of Bridge Periods and Base Shears – Transverse and Longitudinal Directions
b. Frame Analysis and Seismic Design Forces for Piers and Abutments
c. Combined Axial Force and Bending Structural Capacity Check for Piers and Abutments
d. Pile Shear Structural Capacity Check, Minimum Steel, and Pile Connection Details
e. Pier Cap Reinforcement, Connection of Beams to Pier and Abutment Caps, and Support Length

Example 1

3-Span Wide Flange Bridge with Pile Supported Multiple Circular Column Bents, Open Pile Supported Stub Abutments (Pile Bents), and No Skew



Beams W36 x 170; $f'_c = 3500$ psi; $f_y = 36$ ksi; $E_c = 3372$ ksi; $E_s = 29000$ ksi

1. Determination of Bridge Period – Transverse Direction

1.a. Weight of Bridge for Seismic Calculations

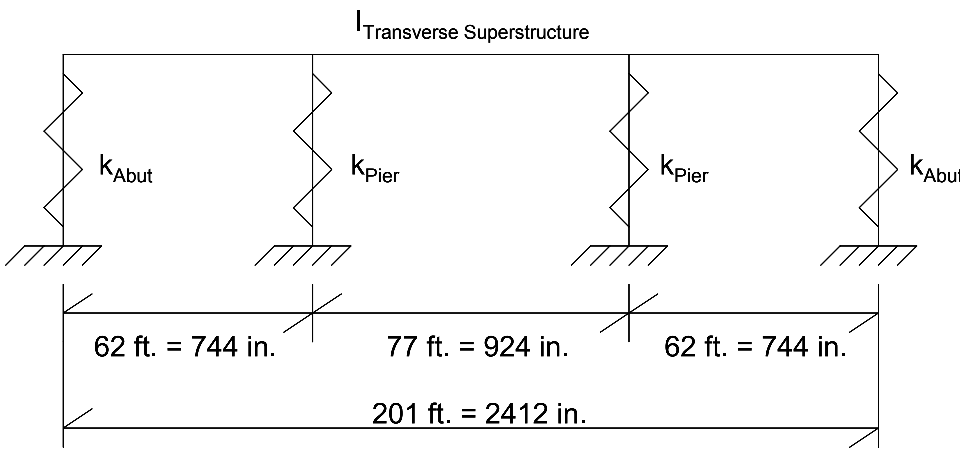
The mass (weight) of a bridge used for seismic design is usually computed first. The mass to consider is that portion of the superstructure and substructures which can reasonably be expected to accelerate horizontally during an earthquake. The total weight of the superstructure including any cross bracing, diaphragms, and parapets should always be included along with the weight of the cap beams and half the columns or walls at piers. In this example, the abutments do not have a reasonable expectation of accelerating to any great degree and are not included. If a bridge has integral abutments, the weight of the end diaphragms should be included (bottom of deck to bottom of bearings). Future wearing surface ("Added Surface" in the figure above) can be added to the weight of a bridge considered for seismic design at the designer's discretion. The total calculated weight can be assumed to act uniformly along the entire length of the superstructure. In more sophisticated analyses, the masses are "lumped" at finite element nodal points.

a.	Beams	W36 X 170		
		Weight per foot 1 beam	=	0.17 k/ft.
		No. of beams	=	7
		Beam weight per foot	=	1.19 k/ft.
b.	Deck	Slab thickness	=	7.5 in. (8.0 in. is Std.)
		Added surface thickness	=	1.5 in.
		Width (Assume added surface extends full deck width)	=	42 ft.
		Deck weight per foot $(7.5+1.5)/12\,{}^{in}/_{ft.} \times 42 \times .15\,{}^{k}/_{ft.^3}$	=	4.725 k/ft.
c.	Parapet	One parapet	=	0.45 k/ft.
		Two parapets	=	0.90 k/ft.
d.	Cross Frames and Bracing	Estimate as 5% of beams	=	0.060 k/ft.
e.	Pier Cap	Length	=	42 ft.
		Width	=	2.5 ft.
		Height	=	4 ft.
		Pier cap weight $42 \times 2.5 \times 4 \times .15\,{}^{k}/_{ft.^3}$	=	63 kips
		Weight of 2 caps	=	126 kips
f.	Columns	Diameter	=	2.5 ft.
		½ Column Height	=	6.25 ft.
		Total No. of Columns	=	8
		Total weight of columns $\pi\left(\frac{2.5}{2}\right)^2 \times 6.25 \times 0.15\,{}^{k}/_{ft.^3} \times 8$	=	36.82 kips

g.	Total Weight	Length of Bridge	=	201 ft.
		a. + b. + c. + d.	=	6.875 k/ft.
		or a. + b. + c. + d.	=	1381.875 kips
		e. + f.	=	162.82 kips
		Total Weight	=	1544.7 kips

1.b. Global Transverse Structural Model of the Bridge

Very simple or more complex global structural models of bridges for dynamic and equivalent static analyses can be used at the designer's discretion. For many typical bridges in Illinois, models that tend to be fairly simple produce reasonable and adequate results for seismic design. Shown below is the global analysis model used for the current example.



Methods for the determination of superstructure, pier and abutment stiffnesses (moment of inertia and spring constants) are given below. At a minimum, the simplest global model should include the stiffnesses of the superstructure and piers with the abutments pinned. Simplification to this level, however, is not recommended but still acceptable. The Uniform Load Method of Analysis, which is advocated by the Department, tends to over estimate reactions at the abutments. Assigning stiffness to the abutments, as opposed to pinned supports that are rigid for transverse displacement, produces more accurate results when using the Uniform Load Method.

The current example bridge can be straightforwardly analyzed with hand methods largely because there are not more than 3 spans and the structure is "very symmetric". For cases where bridges are not symmetric, simple finite element models of the same type shown above can be used for the analysis. If desired, spring finite elements can also be replaced with axial force/truss elements with an equivalent spring stiffness based upon k = (Area)(Mod. of Elasticity)/(Length).

For most cases, the (fundamental) period of the "first mode of vibration" is typically all that needs to be computed because it dominates the dynamic displacement response of a bridge (i.e. multi-modal analysis is not required for typical or regular bridges). The displaced "shape" of the first mode generally approximates that of a half sine wave. This shape will be apparent in subsequent sections below when the fundamental periods of the Example 1 bridge with un-cracked and cracked pier sections are calculated. The second mode of vibration will tend to approximate a full sine wave, the third 1 ½ sine waves, etc. In complex dynamic analyses which use finite elements and time as a variable, the "equations of motion" are coupled together and difficult to solve in the time domain. Transforming the equations of motion and responses of a structure into the frequency domain uncouples them in such a way that the equations become "solvable". Solutions in the frequency domain, at any point in time, essentially can be thought of as giving the relative magnitude of importance each mode has in the total displacement. For many structures, the first mode dominates the solutions and, as such, the responses from higher modes can be neglected for engineering design purposes.

1.c. Transverse Pier Stiffness for Un-cracked and Cracked Columns

For multiple column bent piers, the columns are considered to be the "weak link" or 3rd tier seismic fuse according to IDOT's ERS plan. It should be assumed that the columns deflect in reverse curvature with fixed ends at the bottom of the cap and the top of the crashwall (clear height). The equations below determine the stiffness.

Column Moment of Inertia $$I_c = \frac{\pi\left(\phi_{col}/2\right)^4}{4}\text{; where } \phi_{col} = \text{column diameter}$$

Column Stiffness $$k_c = \frac{12 \times E_c \times I_c}{h_c^3}\text{; where } h_c = \text{clear column height}$$

For a simple analysis, the foundation should be assumed fixed. When the piles are sized, the seismic design forces used will typically be unreduced by an R-Factor (designed elastically). So, the foundation will be significantly stiffer than the columns in the piers.

For the 500 year design earthquake return period event, the columns may be assumed to be "un-cracked". At the 1000 year level, the columns should be considered cracked with an effective moment of inertia of ½ that of I_c. The stiffness of the piers for the current example is given below.

Column Moment of Inertia	$I_c = \dfrac{\pi \times (15\text{ in.})^4}{4} = 39760.8\text{ in.}^4$
Cracked Moment of Inertia	$I_{c/2} = {39760.8\text{ in.}^4}/{2} = 19880.4\text{ in.}^4$
Clear Height of Column	12.5 ft. = 150 in.
Concrete Modulus of Elasticity	E_c = 3372 ksi
Stiffness of Un-cracked Column	$k_c = \dfrac{12 \times 3372 \times 39760.8}{150^3} = 476.7\ {}^{k}/_{in.}$
Stiffness of Cracked Column	$k_c = {476.7\ {}^{k}/_{in.}}/{2} = 238.4\ {}^{k}/_{in.}$
Stiffness of Un-cracked Pier	$k_{Pier} = 476.7\ {}^{k}/_{in.} \times 4\text{ columns} = 1906.8\ {}^{k}/_{in.}$
Stiffness of Cracked Pier	$k_{Pier} = 238.4\ {}^{k}/_{in.} \times 4\text{ columns} = 953.6\ {}^{k}/_{in.}$

Notes for Other Pier Types: Similar calculations to those above can also be used to determine the transverse stiffness of individual column drilled shaft bents, solid wall encased drilled shaft bents, drilled shaft bents with crashwalls, individually encased pile bents, solid wall encased pile bents, solid wall piers supported by a footing and piles, modified hammerhead piers supported by a footing and piles, and trapezoidal multiple column bents with crashwalls. In addition, at the designer's discretion, hammerhead piers may be analyzed as a single column in reverse curvature above ground with similar techniques to those used above.

The "clear height" of individual column drilled shaft bents and individually encased pile bents should be taken from the depth-of-fixity in the soil to the bottom of the cap beam. The clear height for solid wall encased drilled shaft bents and solid wall encased pile bents should be from the depth-of-fixity to the bottom of the solid wall encasement. The walls are assumed to be rigid links in the transverse direction (or a deep cap beam). The clear column height for drilled shaft bents with a crashwall and trapezoidal column bents with crashwall is the same as that for the current example with the drilled shafts below the wall or the piles below the footing assumed fixed. The clear column height of solid wall and modified hammerhead piers supported by a footing and piles should be taken from the depth-of-fixity to the bottom of the footing. The walls and footings are assumed to be rigid links for these two cases. The clear column height for hammerhead piers can be taken from the bottom of the cantilevered cap to the top of the footing at the designer's discretion. Hammerheads can tend to behave somewhat as a single column pier as opposed to a wall.

Methods for dealing with the added complexities of skew, and skew in combination with cross-sections which are not round are given in Examples 2 and 3.

1.d. Transverse Abutment Stiffness

In this example, the stiffness of the abutments will be calculated assuming only the steel H-piles contribute. The piles can be modeled as individual columns in soil in reverse curvature with a clear height extending from the depth-of-fixity in the soil to the bottom of the abutment cap. Batter in the piles for situations such as the current example can be ignored. The designer may also consider the stiffness provided by the abutment and wings bearing on the soil or any other sources of stiffness judged appropriate. For example, integral abutments may be modeled with an additional rotational spring which simulates the stiffness of the diaphragm. For most cases, though, this is not necessary or recommended. The stiffness of the abutments for the current example is given below.

Piles	HP 12 x 74
Number of Piles	9
Weak Axis Pile Moment of Inertia	I_p = 186 in.^{4}
(Typical Orientation for Illinois)	
Steel Modulus of Elasticity	E_s = 29000 ksi
Pile Effective Height	8.0 ft. = 96.0 in.
(from Geotechnical Analysis-	

see also Appendix C)

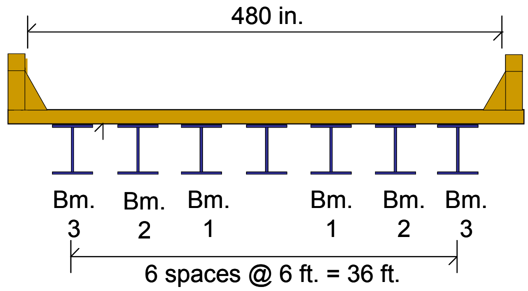
Stiffness of Abutment $$k_{Abut} = \frac{(\text{no. piles}) \times 12 \times E_s \times I_p}{h_p^3}$$

$$k_{Abut} = \frac{9 \times 12 \times 29000 \times 186}{96.0^3} = 658.4 \ {}^{k}\!/\!_{in.}$$

1.e. Transverse Superstructure Stiffness

During an earthquake, the superstructure deflects horizontally as one "effective beam". The deck and beams are the primary contributors to the moment of inertia. The parapets may or may not be considered to contribute to the superstructure moment of inertia. For typical IDOT bridges, it may be most realistic to consider the parapets half effective. Future wearing surface should not be considered to contribute to the superstructure moment of inertia. "Shear lag" in the beams should always be accounted for by considering them half effective. The beam areas "lag" in effectiveness for resisting horizontal loads as the distance from the deck increases. For the current example, the parapets have been assumed to be fully effective. The superstructure moment of inertia calculations are given below.

E_s	29000 ksi
E_c	3372 ksi
n (modular ratio)	8.6
Slab Thickness	7.5 in.
Slab Width	42 ft.
Slab Moment of Inertia	$I_{slab} = \frac{1}{12} \times 7.5 \times \left(42 \text{ ft.} \times 12 \ {}^{in}\!/\!_{ft.}\right)^3 = 8.0 \times 10^7 \text{ in.}^4$
Area of 1 Parapet	432 in.^{2}
Area of 1 Beam	50 in.^{2}
Transformed Beam Area	$A_{steel} = \frac{n \times (\text{Area 1 Beam})}{2 \text{ for Shear Lag}} = \frac{8.6 \times 50}{2} = 215 \text{ in.}^2$



Moment of Inertia of Superstructure Table

	No.	I_0 (In^4)	A (in^2)	$\bar{x}$ (in)	$A \times \bar{x}^2$ (in^4)	I (in^4)
Parapet	2	----	432	240	2.49E+07	4.98E+07
Slab	1	8.00E+07	----	----	----	8.00E+07
Beam 1	2	----	215	72	1.11E+06	2.23E+06
Beam 2	2	----	215	144	4.46E+06	8.92E+06
Beam 3	2	----	215	216	1.00E+07	2.01E+07

I_{Total} 1.610E+08 in^4

1.f. Uniform Load Method Transverse Period Determination for Un-cracked Columns

The first step in the method is to calculate the maximum displacement of the bridge for a simple uniform load, usually 1 k/in. or 1 k/ft. The maximum displacement in this example will occur at the center of the structure. If a bridge has asymmetries such as unequal span lengths, or piers and abutments with different stiffnesses; the maximum deflection will occur somewhere other than the center of the structure. The total uniform load applied to the structure is divided by the maximum deflection to determine an equivalent very simple bridge stiffness. The equivalent stiffness encompasses the effects of the superstructure, abutment and pier stiffnesses. The period is a function of the weight of the structure (determined above) and the equivalent stiffness using a basic equation from structural dynamics. The calculations for the period of the Example 1 bridge are given below with hand methods for piers which are un-cracked. The period for cracked analysis is also given with minimal calculations shown as the method for period determination is the same with different pier stiffnesses.

a. Find the deflection of the bridge for a uniform load of 1 k/in. assuming there are no piers and the abutments are infinitely stiff (deflection of a simple beam).

w (uniform load)	1 k/in.
L (bridge length)	201 ft. = 2412 in.
E_c	3372 in.^{2}
I_{Total}	1.61 x 10^8 in.^{4}

Deflection $$\delta_c = \frac{5 \times w \times L^4}{384 \times E_c \times I_{Total}}$$

$$\delta_c = \frac{5 \times 1 \times 2412^4}{384 \times 3372 \times 161000000} = 0.812 \text{ in.}$$

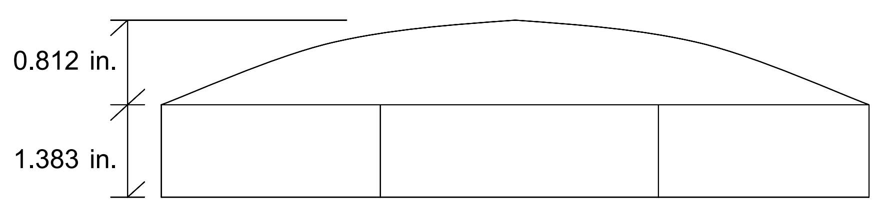
b. Find the deflection of the bridge for a uniform load of 1 k/in. assuming no piers, an infinitely stiff superstructure and abutment springs (simple deflection of a pair of springs).

w (uniform load)	1 k/in.
L (bridge length)	201 ft. = 2412 in.
k_{Abut}	658.4 k/in.

Deflection $$\delta_e = \frac{w \times L}{2} \Big/ k_{Abut}$$

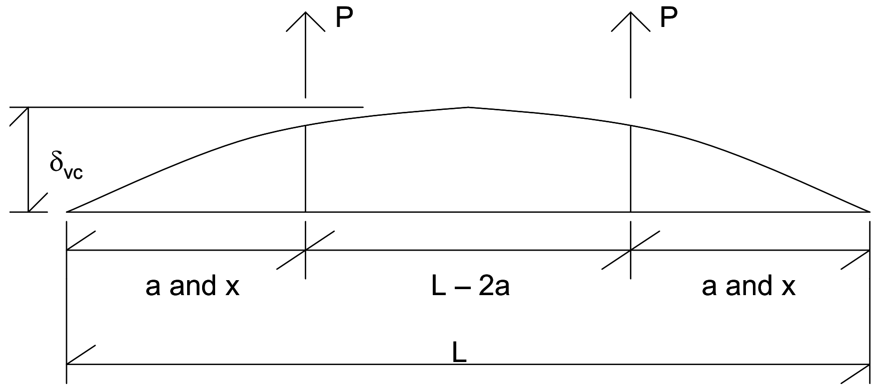
$$\delta_e = \frac{1 \times 2412}{2} \Big/ 658.4 = 1.832 \text{ in.}$$

c. Total deflection for a uniform load of 1 k/in. without piers considered.



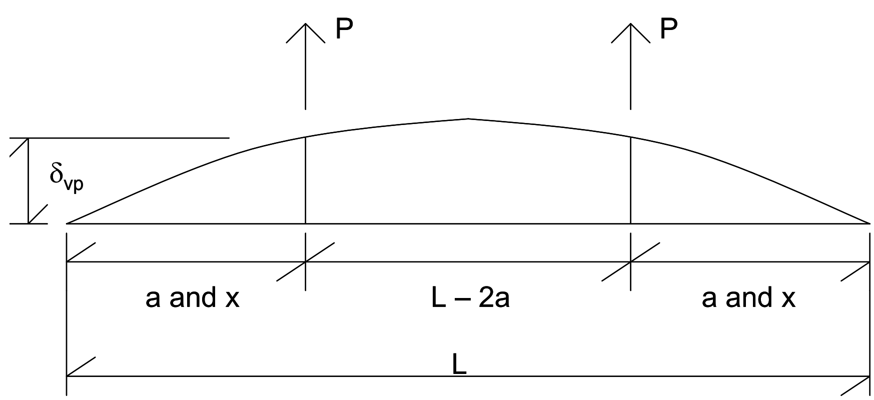
Total Deflection $$\delta_T = \delta_c + \delta_e = 2.644 \text{ in.}$$

d. Find the estimated deflection at the center of the bridge for a point load, P, at each pier location without considering pier stiffness and with infinitely stiff abutments.



L	2412 in.
x	744 in.
a	744 in.
E_c	3372 ksi
I_{Total}	1.61×10^8 in.[4]
Deflection	$\delta_{vc} = \frac{P \times a}{24 \times E_c \times I_{Total}}\left(3 \times L^2 - 4 \times a^2\right)$
	$\delta_{vc} = \frac{P \times 744}{24 \times 3372 \times 161000000}\left(3 \times 2412^2 - 4 \times 744^2\right)$
	$\delta_{vc} = 0.0008702P$

e. Find the estimated deflection at the pier locations for a point load, P, at each pier location without considering pier stiffness and with infinitely stiff abutments.



L	2412 in.
x	744 in.
a	744 in.
E_c	3372 ksi
I_{Total}	1.61×10^8 in.^{4}

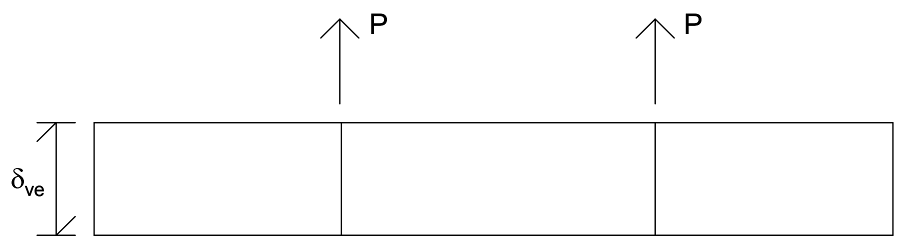
Deflection

$$\delta_{vp} = \frac{P \times x}{6 \times E_c \times I_{Total}}\left(3 \times L \times a - 3 \times a^2 - x^2\right)$$

$$\delta_{vp} = \frac{P \times 744}{6 \times 3372 \times 161000000}\begin{pmatrix}3 \times 2412 \times 744 - \\ 3 \times 744^2 - 744^2\end{pmatrix}$$

$$\delta_{vp} = 0.0007239P$$

f. Find the estimated uniform deflection for a point load, P, at each pier location without considering pier stiffness, with an infinitely stiff superstructure, and with springs at the abutments.



k_{Abut}	658.4 k/in.

Deflection

$$\delta_{ve} = P/k_{Abut} = P/658.4 = 0.0015188P$$

g. Find the ratio (fraction) of the total deflection at the pier locations computed above to the total deflection at center span computed above (steps d., e., and f.).

δ_{vc}	0.0008702P
δ_{vp}	0.0007239P
δ_{ve}	0.0015188P

Fraction (fr) $fr = \frac{\delta_{ve} + \delta_{vp}}{\delta_{ve} + \delta_{vc}} = \frac{0.0015188 + 0.0007239}{0.0015188 + 0.0008702} = 0.9388$

h. Find the pier reactions (V_0) in terms of the actual estimated deflection of the bridge, δ_{max}.

Fraction (fr)	0.9388
k_{Pier}	1906.8 k/in.
Pier Reactions	$V_0 = fr \times \delta_{max} \times k_{Pier} = 0.9388 \times 1906.8 \times \delta_{max}$
	$V_0 = 1790.1\delta_{max}$

i. Solve for δ_{max}

$$\delta_{ve} + \delta_{vc} = 0.002389P$$

Set :

$$P = V_0 = 1790.1\delta_{max}$$

$$\therefore \delta_{ve} + \delta_{vc} = 0.002389 \times 1790.1\delta_{max} = 4.2765\delta_{max}$$

And:

The deflection of the bridge is the actual estimated deflection of the structure without the piers minus that due to the piers.

$$\delta_{max} = \delta_T - 4.2765\delta_{max}$$

$$\delta_{max} = 2.644/5.2765 = 0.501\,\text{in.}$$

j. Solve for the equivalent stiffness of the bridge.

w (uniform load)	1 k/in.
L (bridge length)	201 ft. = 2412 in.
δ_{max}	0.501 in.
Bridge Stiffness	$k_{Bridge} = \frac{w \times L}{\delta_{max}} = \frac{1 \times 2412}{0.501} = 4814.4\ ^{k}/_{in.}$

k. Solve for the period of the bridge.

Total Weight	1544.7 kips
Accel. of Gravity (g)	386.4 in./sec.^{2}
Bridge Stiffness	4814.4 k/in.
Period (T)	$T = 2\pi\sqrt{\frac{W}{g \times k_{Bridge}}} = 2\pi\sqrt{\frac{1544.7}{386.4 \times 4814.4}} = 0.18\ \text{sec.}$

1.g. Uniform Load Method Transverse Period Determination for Cracked Columns

In order to compute the period of the bridge with cracked columns, steps h. through k. from above need only be repeated.

h. Find the pier reactions (V_0) in terms of the actual estimated deflection of the bridge, δ_{max}.

Fraction (fr)	0.9388
k_{Pier}	953.6 k/in.
Pier Reactions	$V_0 = fr \times \delta_{max} \times k_{Pier} = 0.9388 \times 953.6 \times \delta_{max}$
	$V_0 = 895.2\delta_{max}$

i. Solve for δ_{max}.

$$\delta_{ve} + \delta_{vc} = 0.002389P$$

Set :

$$P = V_0 = 895.2\delta_{max}$$

$$\therefore \delta_{ve} + \delta_{vc} = 0.002389 \times 895.2\delta_{max} = 2.1386\delta_{max}$$

And:

The deflection of the bridge is the actual estimated deflection of the structure without the piers minus that due to the piers.

$$\delta_{max} = \delta_T - 2.1386\delta_{max}$$

$$\delta_{max} = 2.644/3.1386 = 0.842 \text{ in.}$$

j. Solve for the equivalent stiffness of the bridge.

w (uniform load)	1 k/in.
L (bridge length)	201 ft. = 2412 in.
δ_{max}	0.842 in.
Bridge Stiffness	$k_{Bridge} = \frac{w \times L}{\delta_{max}} = \frac{1 \times 2412}{0.842} = 2864.6 \text{ k/in.}$

k. Solve for the period of the bridge.

Total Weight	1544.7 kips
Accel. of Gravity (g)	386.4 in./sec.^{2}
Bridge Stiffness	2864.6 k/in.
Period (T)	$T = 2\pi\sqrt{\frac{W}{g \times k_{Bridge}}} = 2\pi\sqrt{\frac{1544.7}{386.4 \times 2864.6}} = 0.23 \text{ sec.}$

When the columns are cracked, the period of the bridge increased from 0.18 sec. to 0.23 sec. or about 28% greater than the un-cracked case. This is because the superstructure stiffness is dominant which is typical of many Illinois bridges. It is also somewhat unusual for the transverse period of a typical bridge in Illinois to be near 1.0 sec. Consequently, if a long (around 0.75 sec. and greater) transverse period is calculated, there may either be an error in the designer's calculations or the bridge may not be modeled properly.

2. Determination of Bridge Period – Longitudinal Direction

2.a. Weight and Global Longitudinal Structural Model of the Bridge

The mass of the bridge used for calculation of the longitudinal period is the same as that calculated above for the transverse direction. For this example, the piers are assumed to be the only elements of the bridge which contribute stiffness to the longitudinal period. The superstructure acts as a rigid link between the two piers with the abutments assumed to provide no resistance to seismic load.

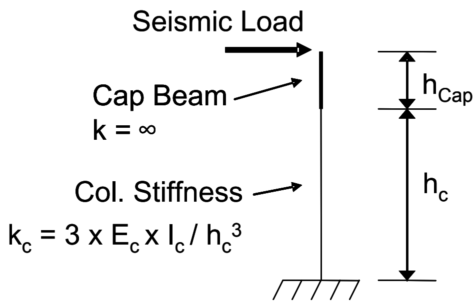
It is also acceptable and/or more correct to consider that the abutments contribute to the stiffness of the bridge in the longitudinal direction depending on the structure configuration. For example, if the abutments are integral, at a minimum, the stiffness of the piles should be part of the longitudinal global model. If the abutments are not integral, the designer may consider the resistance of the beams bearing against a backwall and the soil behind it for one abutment, or the resistance of the piles, or both. It is also acceptable to consider the abutments not contributing to the stiffness in the longitudinal direction even if the bearings are “fixed” but the abutment is an open stub type (pile bent). Making this choice implies that the designer is “relying” upon the piers to a greater extent than the abutments for resistance of seismic forces in the longitudinal direction. If adequate seat widths are provided at the abutments according the 2^{nd} tier of seismic redundancy in IDOT’s ERS strategy, this method just ensures a more conservative pier design in the longitudinal direction.

2.b. Longitudinal Pier Stiffness for Un-cracked and Cracked Columns

The columns for multiple circular column bents with cap beams should be assumed to deflect as cantilevers which deform from fixed ends at the top of the crashwall to the bottom of the cap. The rigid body rotation of the cap should also be included in the stiffness determination. The equations and derivation below determine the stiffness, and the figure provides an illustration for guidance.

Column Moment of Inertia $$I_c = \frac{\pi\left(\phi_{col}/2\right)^4}{4}\text{; where } \phi_{col} = \text{column diameter}$$

Column Stiffness w/o Cap $k_c = \frac{3 \times E_c \times I_c}{h_c^3}$; where h_c = clear column height



Column Stiffness w/ Cap

For a load P, the deflection at the top of a column, δ_{TC}, is:

$$k_c \times \delta_{TC} = P \Rightarrow \delta_{TC} = P/k_c = \frac{P \times h_c^3}{3 \times E_c \times I_c}$$

For a load P, the rotation at the top of a column, θ_{TC}, is:

$$\theta_{TC} = \frac{P \times h_c^2}{2 \times E_c \times I_c}$$

The added deflection of the pier at the top of the cap beam, δ_A, is:

$$\delta_A = h_{Cap} \times \theta_{TC}$$

The total deflection, δ_{TD}, is $\delta_{TC} + \delta_A$ and the final pier column long. stiffness then is:

$$k_{Long\,Pier\,per\,Column} = P/\delta_{TD}$$

For a simple analysis, the foundation should be assumed fixed. When the piles are sized, the seismic design forces used will typically be unreduced by an R-Factor (designed elastically). So, the foundation will be significantly stiffer than the columns in the piers. The crashwall does deflect, but this effect can be ignored for the determination of the longitudinal period.

For the 500 year design earthquake return period event, the columns may be assumed to be "un-cracked". At the 1000 year level, the columns should be considered cracked with an effective moment of inertia of ½ that of I_c. The stiffness of the piers for the current example is given below.

Column Moment of Inertia	$I_c = \frac{\pi \times (15\text{ in.})^4}{4} = 39760.8\text{ in.}^4$
Cracked Moment of Inertia	$I_{c/2} = 39760.8\text{ in.}^4 / 2 = 19880.4\text{ in.}^4$
Clear Height of Column	12.5 ft. = 150 in.
Concrete Modulus of Elasticity	E_c = 3372 ksi
Stiffness of Un-cracked Column	$k_c = \frac{3 \times 3372 \times 39760.8}{150^3} = 119.2\ ^k/_{in.}$
Stiffness of Cracked Column	$k_c = 119.2\ ^k/_{in.} / 2 = 59.6\ ^k/_{in.}$
Stiffness of Un-cracked Column w/ Cap	δ_{TC} deflection at top of column for a load P :

$$k_c \times \delta_{TC} = P \Rightarrow \delta_{TC} = P/119.2 = 0.008389P$$

θ_{TC}, rotation at top of column for a load P :

$$\theta_{TC} = \frac{P \times 150^2}{2 \times 3372 \times 39760.8} = 0.00008391P$$

δ_A, the added deflection at the top of cap :

$$\delta_A = h_{Cap} \times \theta_{TC} = 48\text{ in.} \times 0.00008391P = 0.004028P$$

δ_{TD}, the total deflection :

$$\delta_{TD} = \delta_{TC} + \delta_A = 0.008389P + 0.004028P = 0.01242P$$

The column stiffness with cap is :

$$k_{Long\ Pier\ per\ Column} = P/\delta_{TD} = 80.5\ ^k/_{in.}$$

Stiffness of Un-cracked Pier	$k_{Pier} = 80.5\ ^k/_{in.} \times 4\text{ columns} = 322.0\ ^k/_{in.}$
Stiffness of Cracked Pier	$k_{Pier} = 322.0\ ^k/_{in.} / 2 = 161.0\ ^k/_{in.}$

Notes for Other Pier Types: Similar calculations to those above can also be used to determine the longitudinal stiffness of individual column drilled shaft bents, solid wall encased drilled shaft bents, drilled shaft bents with crashwalls, individually encased pile bents, solid wall encased pile bents, solid wall piers supported by a footing and piles, hammerhead and modified hammerhead piers supported by a footing and piles, and trapezoidal multiple column bents with crashwalls.

The “clear height” of individual column drilled shaft bents and individually encased pile bents should be taken from the depth-of-fixity in the soil to the bottom of the cap beam. The clear height for solid wall encased drilled shaft bents and solid wall encased pile bents should be from the depth-of-fixity to the bottom of the solid wall encasement. The walls are assumed to be rigid links in the longitudinal direction (or a deep cap beam which rotates significantly). The clear column height for drilled shaft bents with a crashwall and trapezoidal column bents with crashwall is the same as that for the current example with the drilled shafts below the wall or the piles below the footing assumed fixed. The clear column height of solid wall, hammerhead and modified hammerhead piers supported by a footing and piles should be taken from the top of the footing to the bottom of the cap as appropriate. The walls are treated as one large column bending about the weak axis with the foundations assumed fixed.

2.c. Uniform Load Method Longitudinal Period Determination for Un-cracked and Cracked Columns

For the current example, the Uniform Load Method can be used in a more straightforward manner than for the transverse case to calculate the longitudinal bridge period with the equation given below.

$$T = 2\pi\sqrt{\frac{M}{\text{No. of Piers} \times k_{pier}}}\text{; where } M = \text{mass of bridge}$$

If a bridge has unequal pier and/or abutment stiffnesses, the total stiffness for all substructure elements considered should be substituted in the denominator of the equation above. Bridge periods for the current example with un-cracked and cracked columns are given below.

Mass of Bridge $M = \dfrac{\text{Weight of Bridge}}{g} = \dfrac{1544.7}{386.4} = 3.998\ ^{k-sec.^2}\!/_{in.}$

Period with Un-Cracked Columns $T = 2\pi\sqrt{\dfrac{3.998}{2 \times 322.0}} = 0.50$ sec.

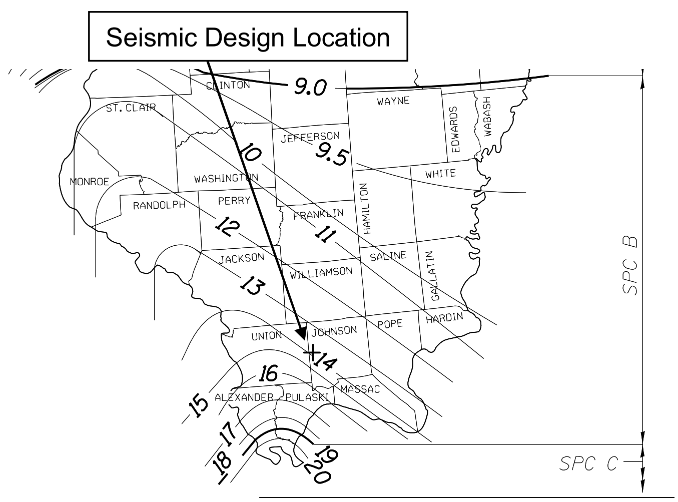
Period with Cracked Columns $T = 2\pi\sqrt{\dfrac{3.998}{2 \times 161.0}} = 0.70$ sec.

When the columns are cracked, the period of the bridge increased from 0.50 seconds to 0.70 seconds or about 40% greater than the un-cracked case. This is because the superstructure stiffness does not play a role in the fundamental period except as a link to the substructures which is typical of most Illinois bridges. It is also typical for the longitudinal period of bridges without significant skew to be a fair amount larger than the transverse period.

3. Determination of Base Shears – 500 Year Design Earthquake Return Period

<u>3.a. Design Response Spectrum (LFD)</u>

Acceleration Coefficient, A	0.14g (See Below)
Seismic Performance Category	B $(0.09 < A \leq 0.19)$
Importance Category	Essential
Soil Profile Type	II
Site Coefficient, S	1.2



Transverse Direction for the Uncracked Column Case:

$$C_s = \frac{1.2A \times S}{T^{2/3}} = \frac{1.2 \times 0.14 \times 1.2}{0.18^{2/3}} = 0.63 > 2.5A = .35$$

$$\therefore \text{Use } 0.35$$

Values greater than 2.5A are expected in the transverse direction for many if not most typical bridges with short to medium height columns built in Illinois.

Longitudinal Direction for the Uncracked Column Case:

$$C_s = \frac{1.2A \times S}{T^{2/3}} = \frac{1.2 \times 0.14 \times 1.2}{0.50^{2/3}} = 0.32 < 2.5A = .35$$

$$\therefore \text{Use } 0.32$$

Values less than 2.5A are expected in the longitudinal direction for many if not most typical bridges which are modeled without a contribution from the abutments.

<u>3.b. Transverse Base Shear</u>

Total Base Shear for the Bridge = $C_s \times \text{Wt. of Bridge} = 0.35 \times 1544.7 = 540.6 \text{ kips}$

Or $540.6 \text{ kips} / 2412 \text{ in.} = 0.224 \text{ k/in.}$

The transverse seismic base shear at the piers ($V_{\text{Base Shear P (T)}}$) can be determined as the ratio of the uniform base shear load calculated above (0.224 k/in.) to the applied uniform load from the period calculations (1 k/in.) times the deflection at the center of the bridge for a 1 k/in. load (0.501 in.) times the stiffness of a pier in relation to the deflection at the center of the structure ($1790.1\delta_{max}$).

$$V_{\text{Base Shear P (T)}} = \frac{0.224}{1} \times 0.501 \times 1790.1 = 200.9 \text{ kips}$$

The transverse seismic base shear at the abutments ($V_{Base\ Shear\ A\ (T)}$) is calculated from statics as the total base shear (540.6 kips) divided by 2 minus the base shear at a pier (190.6 kips)

$$V_{Base\ Shear\ A\ (T)} = \frac{540.6}{2} - 200.9 = 69.4\ \text{kips}$$

3.c. Longitudinal Base Shear

$$\text{Total Base Shear for the Bridge} = C_s \times \text{Wt. of Bridge} = 0.32 \times 1544.7 = 494.3\ \text{kips}$$

For this example the longitudinal seismic base shear ($V_{Base\ Shear\ P\ (L)}$) is distributed equally to each pier. If the pier stiffnesses were unequal, the base shear would be distributed according to the relative stiffness magnitudes of each pier.

$$V_{Base\ Shear\ P\ (L)} = \frac{494.3}{2} = 247.2\ \text{kips}$$

The longitudinal seismic base shear at the abutments ($V_{Base\ Shear\ A\ (L)}$) is zero.

$$V_{Base\ Shear\ A\ (L)} = 0$$

4. Determination of Base Shears – 1000 Year Design Earthquake Return Period

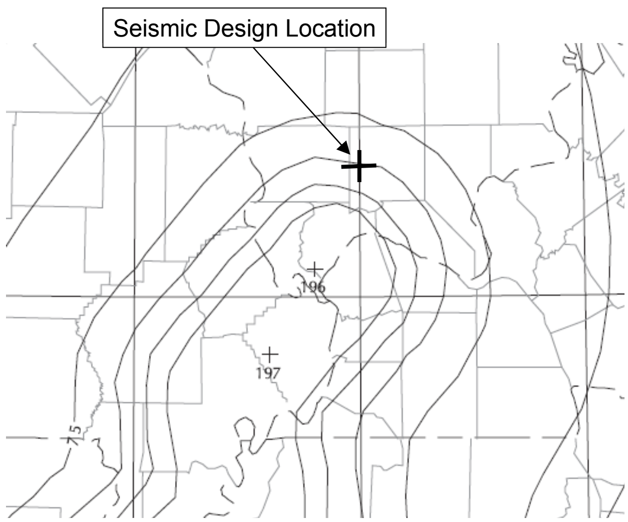
4.a. Design Response Spectrum (LRFD)

Reference Appendix 3.15.A of the Bridge Manual and the LRFD Code for more information on the formulation of the 1000 yr. Design Response Spectrum.

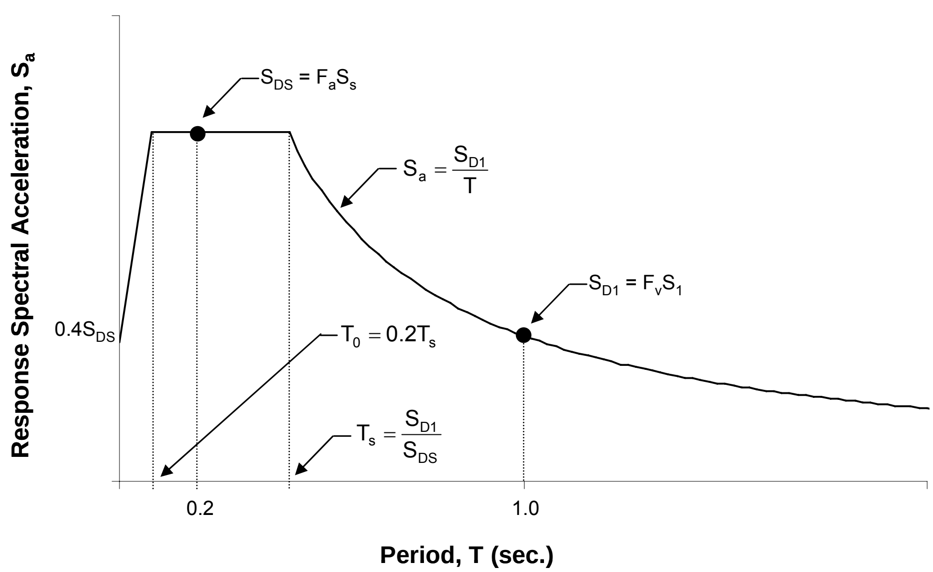
S_s (Short Period Acceleration)	1.035g (See Below)
S_1 (1-Sec. Period Acceleration)	0.259g (Map Not Shown)
Soil Type	Class D (In Upper 100 ft. of Soil Profile)
F_a (Short Period Soil Coef.)	1.09
F_v (1-sec. Period Soil Coef.)	1.88

S_{DS}	$F_a S_s = 1.09 \times 1.035 = 1.128g$
S_{D1}	$F_v S_1 = 1.88 \times 0.259 = 0.487g$
Seismic Performance Zone	$3 \ (0.3 < F_v S_1 \leq 0.5$ BM Table 3.15.2-1)
Importance Category	Essential

Short period, 0.2 sec., design acceleration map (circa 2005, 2008 LRFD map similar) and seismic design location (same location as for 500 yr. design return period earthquake).



Definitions and a graphical representation of the design response spectrum (with approximate acceleration at zero sec. period).



T_s = $0.487/1.128 = 0.432$ sec.

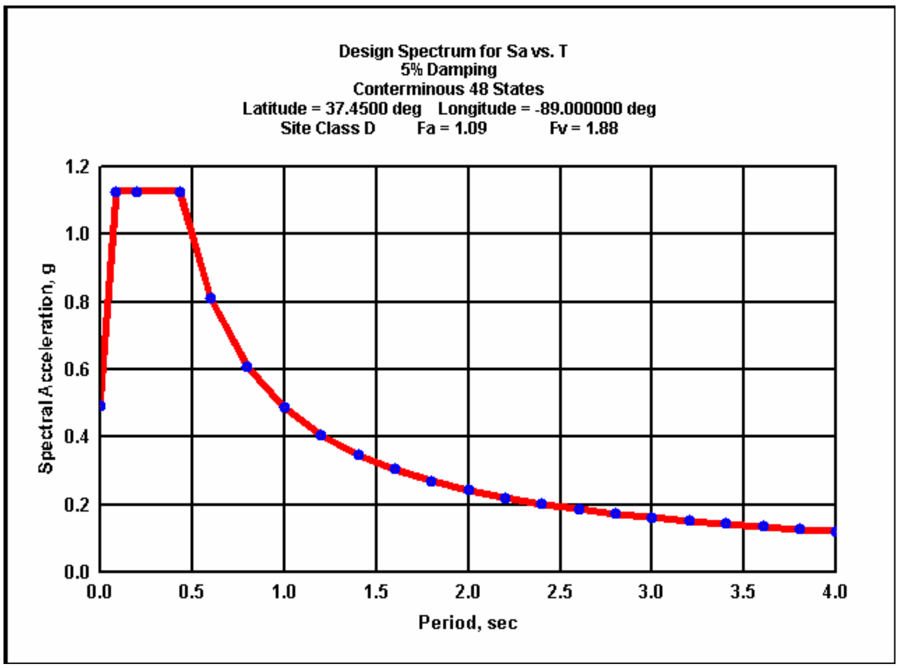
T_0 = $0.2 \times 0.432 = 0.086$ sec.

Less than T_0, S_a = $0.6\frac{S_{DS}}{T_0}T + 0.4S_{DS} = 7.87T + 0.4512$

Greater than T_s, S_a = $0.487/T$

(Where T = T_m, and S_a = C_{sm} in the LRFD Code)

Plot of the design response spectrum.



Transverse Direction for the Cracked Column Case:

S_a (C_{sm} in the LRFD Code) = 1.128 (T = 0.23 < T_s = 0.432 sec.)

Values at the "plateau" (analogous to 2.5A in the LFD Code) are expected in the transverse direction for many if not most typical bridges with short to medium height columns built in Illinois.

Longitudinal Direction for the Cracked Column Case:

S_a (C_{sm} in the LRFD Code) = $0.487/0.7 = 0.7$ (T = 0.7 > T_s = 0.432 sec.)

Values past the plateau (analogous to 2.5A in the LFD Code) are expected in the longitudinal direction for many if not most typical bridges which are modeled without a contribution from the abutments.

<u>4.b. Transverse Base Shear</u>

Total Base Shear = $S_a \times \text{Wt. of Bridge} = 1.128 \times 1544.7 = 1742.4 \text{ kips}$

Or $1742.4 \text{ kips} / 2412 \text{ in.} = 0.722 \text{ k/in.}$

The transverse seismic base shear at the piers ($V_{Base\ Shear\ P\ (T)}$) can be determined as the ratio of the uniform base shear load calculated above (0.722 k/in.) to the applied uniform load from the period calculations (1 k/in.) times the deflection at the center of the bridge for a 1 k/in. load (0.842 in.) times the stiffness of a pier in relation to the deflection at the center of the structure ($895.2\delta_{max}$).

$V_{Base\ Shear\ P\ (T)}$ = $\frac{0.722}{1} \times 0.842 \times 895.2 = 544.2 \text{ kips}$

The transverse seismic base shear at the abutments ($V_{Base\ Shear\ A\ (T)}$) is calculated from statics as the total base shear (1742.4 kips) divided by 2 minus the base shear at a pier (503.2 kips)

$V_{Base\ Shear\ A\ (T)}$ = $\frac{1742.4}{2} - 544.2 = 327.0 \text{ kips}$

<u>4.c. Longitudinal Base Shear</u>

Total Base Shear = $S_a \times \text{Wt. of Bridge} = 0.70 \times 1544.7 = 1081.3 \text{ kips}$

For this example the longitudinal seismic base shear ($V_{Base\ Shear\ P\ (L)}$) is distributed equally to each pier. If the pier stiffnesses were unequal, the base shear would be distributed according to the relative stiffness magnitudes of each pier.

$V_{Base\ Shear\ P\ (L)}$ = $\frac{1081.3}{2} = 540.7\ \text{kips}$

The longitudinal seismic base shear at the abutments ($V_{Base\ Shear\ A\ (L)}$) is zero.

$V_{Base\ Shear\ A\ (L)}$ = 0

There are significant differences in the seismic base shears between the 500 and 1000 yr. design earthquakes. The total base shear in the transverse direction was elevated from 540.6 to 1742.4 kips (222% increase) and in the longitudinal direction it was elevated from 494.3 to 1081.3 kips (119% increase), respectively. The effect of the total increase in base shear in the transverse direction, however, was mitigated because of the "redistribution" of reactions to the abutments due to the assumption that the piers will crack during a significant seismic event. At the piers, the transverse base shear increased from 200.9 to 544.2 kips (171%), while at the abutments it increased from 69.4 to 327.0 kips (371%). There is no redistribution of forces in the longitudinal direction for this bridge.

5. Frame Analysis and Columnar Seismic Forces for Multiple Column Bent – 500 and 1000 Year Design Earthquake Return Period

5.a. Pier Forces – Dead Load

Dead Load of Superstructure

(Use Wt. from Previous Calculations) = 1544.7 kips

Bridge Length = 201 ft.

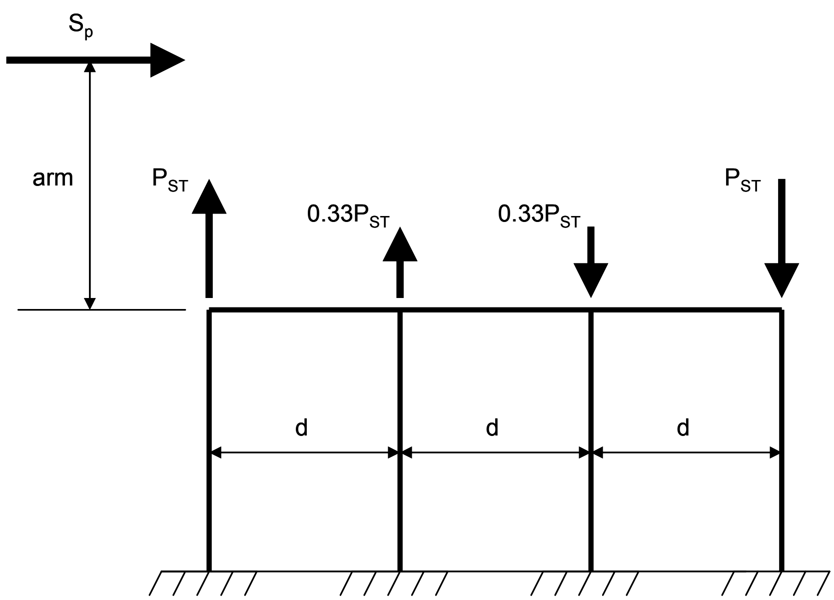
Dead Load Per ft. of Bridge = $1544.7/201 = 7.685\ \text{k/ft.}$

Dead Load Per Pier (Use Statics) = $7.685 \times \left(\frac{5}{8} L_{OuterSpan} + \frac{1}{2} L_{CenterSpan}\right)$

	=	$7.685 \times \left(\frac{5}{8} \times 62 + \frac{1}{2} \times 77\right) = 593.7\text{ kips}$
No. of Columns Per Pier	=	4
Dead Load Per Column	=	$593.7/4 = 148.4\text{ kips}$
Added Dead Load for Bot. Half of 1 Col.		
Not Considered in Pt. 1.a. Sub-Pt. f.	=	$36.82\text{ kips}/8\text{ Columns in Bridge} = 4.60\text{ kips}$
Design Dead Load Per Column	=	148.4 + 4.6 = 153.0 kips

5.b. Pier Forces – Transverse Overturning

The seismic base shear at each pier theoretically acts through the centroid of the superstructure. It is acceptable to assume/approximate that the centroid acts at the center of the deck. From statics, shown below, an “overturning moment” produces axial compression and tension across the bent. The seismic base shear also produces “frame action” forces in the columns of the bent and acts at the top of the columns (the “eccentricity” or “arm” of the base shear is taken into account through consideration of the overturning moment) Frame action force analysis is given in the following section. Appendix A contains overturning moment solutions for bents with 2 to 13 columns.



$$M = S_p \times arm$$
$$M = 2\left(P_{ST} \times \tfrac{3}{2} d\right) + 2\left(\tfrac{1}{3} P_{ST} \times \tfrac{1}{2} d\right)$$
$$\therefore P_{ST} = \frac{3}{10} M/d$$

500 year return period:

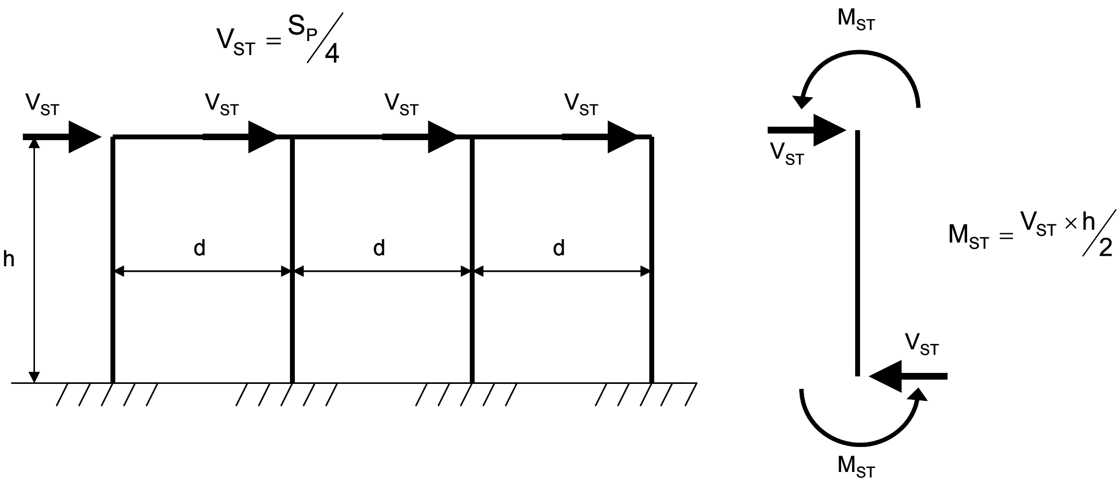
S_p (Base Shear at Pier)	=	200.9 kips
arm (Base Shear Eccentricity)		
Cap Height + Bearing Height + Beam		
Height+ ½ Deck Thickness	=	$4 + 0.5 + 3 + \frac{1}{2}\left(7.5/12\right) = 7.8125$ ft.
d (Center-to-Center Col. Distance)	=	12 ft.
M (Overturning Moment)	=	$200.9 \times 7.8125 = 1569.5$ k - ft.
P_{ST} (Maximum Axial Columnar Force)	=	$\frac{3}{10} 1569.5/12 = 39.2$ kips

1000 year return period:

S_p (Base Shear at Pier)	=	544.2 kips
M (Overturning Moment)	=	$544.2 \times 7.8125 = 4251.6$ k - ft.
P_{ST} (Maximum Axial Columnar Force)	=	$\frac{3}{10} 4251.6/12 = 106.3$ kips

<u>5.c. Pier Forces – Transverse Frame Action</u>

Taking account of the overturning moment "transfers" the seismic base shear at the pier to the tops of the columns. This shear produces moments, shears and axial forces in each column of the bent through "frame action." Free body diagram solutions for these seismic forces are shown below. The determination of moment and shear in each column is more straightforward than for axial force. The simple solutions for moment in the columns are very accurate while conservative solutions for axial force due frame action are emphasized for the critical outside columns. Appendix B contains frame action columnar axial force solutions for bents with 2 to 6 or more columns.

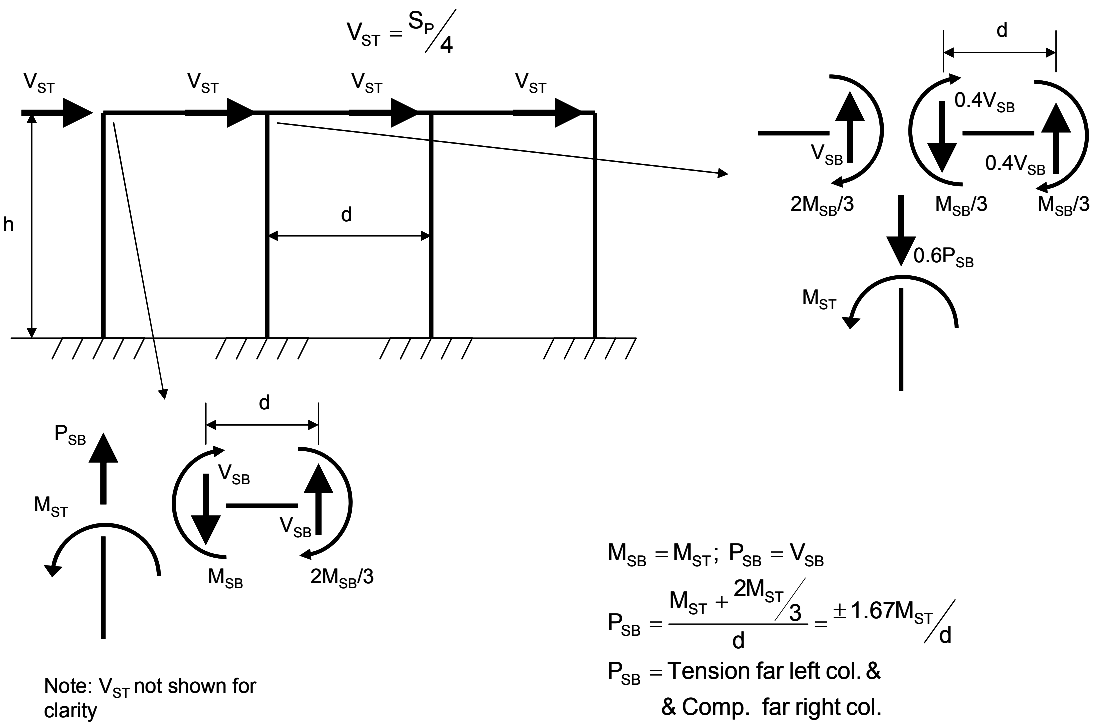


500 Year Return Period:

S_p (Base Shear at Pier)	=	200.9 kips
Column Height (Clear)	=	12.5 ft.
V_{ST} (Shear Per Column)	=	$200.9/4 = 50.2$ kips
M_{ST} (Moment Per Column)	=	$50.2 \times 12.5/2 = 313.8$ k - ft.

1000 Year Return Period:

S_p (Base Shear at Pier)	=	544.2 kips
V_{ST} (Shear Per Column)	=	$544.2/4 = 136.1$ kips
M_{ST} (Moment Per Column)	=	$136.1 \times 12.5/2 = 850.6$ k - ft.



500 Year Return Period:

M_{ST} (Moment Per Column)	=	313.8 k-ft.
d (Center-to-Center Col. Distance)	=	12 ft.
P_{SB} (Maximum Axial Columnar Force)	=	$1.67 \times 313.8/12 = 43.7$ kips

1000 Year Return Period:

M_{ST} (Moment Per Column)	=	850.6 k - ft.
P_{SB} (Maximum Axial Columnar Force)	=	$1.67 \times 850.6/12 = 118.4$ kips

5.d. Pier Forces – Longitudinal Cantilever

Only simple cantilever statics is required to determine the seismic shear and moment in the longitudinal direction.

500 Year Return Period:

S_L (Base Shear at Pier)	=	247.2 kips

Column Height (Clear)	=	12.5 ft.
Cap Beam Height	=	4.0 ft.
V_{SL} (Shear Per Column)	=	$247.2 / 4 = 61.8$ kips
$M_{ColBot(SLB)}$ (Moment Per Column)	=	$61.8 \times (12.5 + 4) = 1019.7$ k - ft.

1000 Year Return Period:

S_L (Base Shear at Pier)	=	540.7 kips
V_{SL} (Shear Per Column)	=	$540.7 / 4 = 135.2$ kips
$M_{ColBot(SLB)}$ (Moment Per Column)	=	$135.2 \times (12.5 + 4) = 2230.8$ k - ft.

6. *Seismic Design Forces for Multiple Column Bent Including R-Factor, P-Δ, and Combination of Orthogonal Forces – 500 and 1000 Year Design Earthquake Return Period*

6.a. R-Factor

R-Factors should only be used to reduce the moments calculated from the base shears of an "elastic" analysis as was conducted above. As recommended in the Bridge Manual (Section 3.15.4.4.3) and the LRFD Code for "Essential Bridges" an R-Factor of 3.5 will be used for the bent in this example. LFD Div. I-A recommends a value of 5 for this bent type. The R-Factor tables in LRFD and LFD have some differences which have the potential to cause confusion. There have also been questions over the years about how the described bridge types in the R-Factor tables in both LRFD and LFD "fit" with actual Illinois bridges in practice. Section 3.15.4.4 of the Bridge Manual attempts to answer some of these questions by providing specific recommendations for R-Factors for a number of common pier types built in Illinois.

6.b. P-Δ

Exacting methods for determining amplification of bending moments for P-Δ effects is not considered overly significant by the Department in most cases for seismic design of bridges. For bents of the type in this example, which have a relatively short clear column height (10 to 15 ft.), the amplification can be estimated as 5% for both the transverse and longitudinal directions. In the range of 15 to 20 ft., the amplification may be estimated as 10%. For greater heights, P-Δ effects may either be calculated or estimated by adding 5% for each 5 ft. increment above 20 ft. clear height. The estimates given above also apply to multiple column drilled shaft bents with crashwall.

Columns in individually encased piles bents and individual drilled shaft bents tend to have longer "effective clear heights" extending from the bottom of the cap to the depth-of-fixity. The methods given above for estimating P-Δ effects for these bent types are permitted at the discretion of the designer.

P-Δ effects should not be considered for walls, hammerheads, modified hammerheads, solid wall encased pile bents, solid wall encased drilled shaft bents, and piles analyzed and designed as columns.

6.c. Summary and Combination of Orthogonal Column Forces Used for Design

The forces on the two exterior columns in the example bridge are focused on for design because they experience the most extreme earthquake forces. The pier columns should be designed for the possibility of earthquake accelerations which can be in opposite transverse directions and opposite longitudinal directions. They are also required to be designed for the cases "mostly longitudinal and some transverse accelerations" (Longitudinal Dominant – Load Case 1) and "some longitudinal and mostly transverse accelerations" (Transverse Dominant – Load Case 2).

Since the bridge in this example has round columns and is not skewed, the summary and combination of orthogonal forces used for design is straightforward. More complex cases with skew and non-round columns are considered in subsequent examples in this design guide. The equations below present the basic method for combination of orthogonal forces.

Load Case 1 (Longitudinal Dominant)

$$V_z^D = 1.0\left|V_z^L\right| + 0.3\left|V_z^T\right|$$

Load Case 2 (Transverse Dominant)

$$V_z^D = 0.3\left|V_z^L\right| + 1.0\left|V_z^T\right|$$

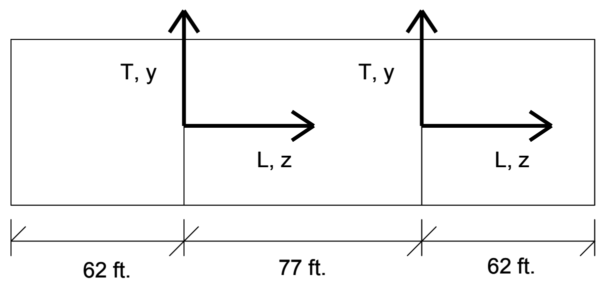
$$V_y^D = 1.0\left|V_y^L\right| + 0.3\left|V_y^T\right| \qquad V_y^D = 0.3\left|V_y^L\right| + 1.0\left|V_y^T\right|$$

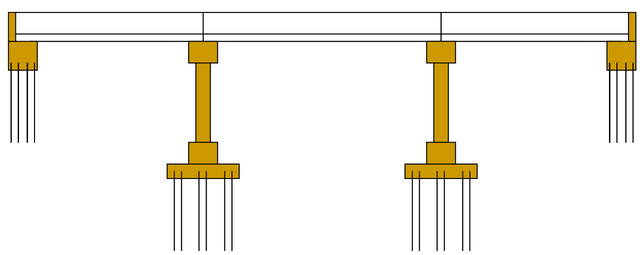
$$M_z^D = 1.0\left|M_z^L\right| + 0.3\left|M_z^T\right| \qquad M_z^D = 0.3\left|M_z^L\right| + 1.0\left|M_z^T\right|$$

$$M_y^D = 1.0\left|M_y^L\right| + 0.3\left|M_y^T\right| \qquad M_y^D = 0.3\left|M_y^L\right| + 1.0\left|M_y^T\right|$$

$$P^D = 1.0\left|P^L\right| + 0.3\left|P^T\right| \qquad P^D = 0.3\left|P^L\right| + 1.0\left|P^T\right|$$

For the bridge in this example the Longitudinal- and z-axes, and Transverse- and y-axes coincide as shown below.





Shown below are the Load Case 1 and Load Case 2 forces used for seismic design with R-Factor (3.5), P-Δ amplification (1.05), and axial dead load (153 kips) effects all considered for the 500 and 1000 year design return period earthquakes.

500 Year Return Period – Load Case 1 – Longitudinal Dominant (per column):

$$V_z^D = 1.0\left|V_z^L\right| + 0.3\left|V_z^T\right| = 1.0\left|V_{SL}\right| + 0.3\left|0\right| = 1.0\left|61.8\right| = 61.8 \text{ kips}$$

$$V_y^D = 1.0\left|V_y^L\right| + 0.3\left|V_y^T\right| = 1.0\left|0\right| + 0.3\left|V_{ST}\right| = 0.3\left|50.2\right| = 15.1 \text{ kips}$$

$$M_z^D = 1.0\left|M_z^L\right| + 0.3\left|M_z^T\right| = 1.0\left|0\right| + 0.3\left|1.05 \times M_{ST} / 3.5\right| = 0.3\left|1.05 \times 313.8 / 3.5\right| = 28.2 \text{ k-ft.}$$

$$M_y^D = 1.0\left|M_y^L\right| + 0.3\left|M_y^T\right| = 1.0\left|\frac{1.05 \times M_{SLB}}{3.5}\right| + 0.3\left|0\right| = 1.0\left|\frac{1.05 \times 1019.7}{3.5}\right| = 305.9 \text{ k - ft.}$$

$$P^D = 1.0\left|P^L\right| + 0.3\left|P^T\right| \rightarrow P^D = P_{Dead} + 1.0\left|0\right| \pm 0.3\left|P_{ST} + P_{SB}\right| = 153 \pm 0.3\left|39.2 + 43.7\right| =$$
$$= 128.1 \text{ and } 177.9 \text{ kips}$$

Note that the Department recommends a load factor of 1.0 be used for dead loads for LRFD and LFD design.

The design shears and moments can be added (as vectors) since the columns are round in order to further simplify the design forces.

$$V^D = \sqrt{61.8^2 + 15.1^2} = 63.6 \text{ kips}$$

$$M^D = \sqrt{28.2^2 + 305.9^2} = 307.2 \text{ k - ft.}$$

$$P^D = 128.1 \text{ and } 177.9 \text{ kips}$$

500 Year Return Period – Load Case 2 – Transverse Dominant (per column):

$$V_z^D = 0.3\left|V_z^L\right| + 1.0\left|V_z^T\right| = 0.3\left|V_{SL}\right| + 1.0\left|0\right| = 0.3\left|61.8\right| = 18.5 \text{ kips}$$

$$V_y^D = 0.3\left|V_y^L\right| + 1.0\left|V_y^T\right| = 0.3\left|0\right| + 1.0\left|V_{ST}\right| = 1.0\left|50.2\right| = 50.2 \text{ kips}$$

$$M_z^D = 0.3\left|M_z^L\right| + 1.0\left|M_z^T\right| = 0.3\left|0\right| + 1.0\left|\frac{1.05 \times M_{ST}}{3.5}\right| = 1.0\left|\frac{1.05 \times 313.8}{3.5}\right| = 94.1 \text{ k - ft.}$$

$$M_y^D = 0.3\left|M_y^L\right| + 1.0\left|M_y^T\right| = 0.3\left|\frac{1.05 \times M_{SLB}}{3.5}\right| + 1.0\left|0\right| = 0.3\left|\frac{1.05 \times 1019.7}{3.5}\right| = 91.8 \text{ k - ft.}$$

$$P^D = 0.3\left|P^L\right| + 1.0\left|P^T\right| \rightarrow P^D = P_{Dead} + 0.3\left|0\right| \pm 1.0\left|P_{ST} + P_{SB}\right| = 153 \pm 1.0\left|39.2 + 43.7\right| =$$
$$= 70.1 \text{ and } 235.9 \text{ kips}$$

Further simplification of the design shears and moments leads to the following.

$$V^D = \sqrt{18.5^2 + 50.2^2} = 53.5 \text{ kips}$$

$$M^D = \sqrt{94.1^2 + 91.8^2} = 131.5 \text{ k - ft.}$$

$$P^D = 70.1 \text{ and } 235.9 \text{ kips}$$

1000 Year Return Period – Load Case 1 – Longitudinal Dominant (per column):

The same form of the calculations above leads to the following simplified design forces.

$$V^D = \sqrt{135.2^2 + 40.8^2} = 141.2 \text{ kips}$$

$$M^D = \sqrt{669.2^2 + 76.6^2} = 673.6 \text{ k-ft.}$$

$$P^D = 85.6 \text{ and } 220.4 \text{ kips}$$

1000 Year Return Period – Load Case 2 – Transverse Dominant (per column):

The same form of the calculations above leads to the following simplified design forces.

$$V^D = \sqrt{40.6^2 + 136.1^2} = 142.0 \text{ kips}$$

$$M^D = \sqrt{200.8^2 + 255.2^2} = 324.7 \text{ k-ft.}$$

$$P^D = -71.7 \text{ and } 377.7 \text{ kips (Negative indicates tension)}$$

7. Column Design Including Overstrength Plastic Moment Capacity – 500 and 1000 Year Design Earthquake Return Period

7.a. Column Design for Axial Force and Moment

Since the columns in the example pier bents are round, a simple uni-axial bending – axial force interaction diagram formulation can be used for design and compared with the forces calculated above. For the 500 year earthquake return period, the example bridge is in Category B. As such, the ϕ factor (strength reduction factor) used for design should be 0.75. For the 1000 year earthquake return period, the example bridge is located in Zone 3 and the ϕ factor is 1.0 according to the recommendation in Section 3.15.4.4.1 of the Bridge Manual.

500 Year Return Period Column Design:

In order to compare the design axial forces and moments calculated above to a "nominal" or unreduced column interaction diagram, they should be divided by the ϕ factor (0.75). These computations are shown below.

Load Case 1 – Longitudinal Dominant

$$M^D/\phi = 307.2/0.75 = 409.6 \text{ k-ft.}$$

$$P^D/\phi = 128.1/0.75 = 170.8 \text{ kips}$$

$$P^D/\phi = 177.9/0.75 = 237.2 \text{ kips}$$

Load Case 2 – Transverse Dominant

$$M^D/\phi = 131.5/0.75 = 175.3 \text{ k-ft.}$$

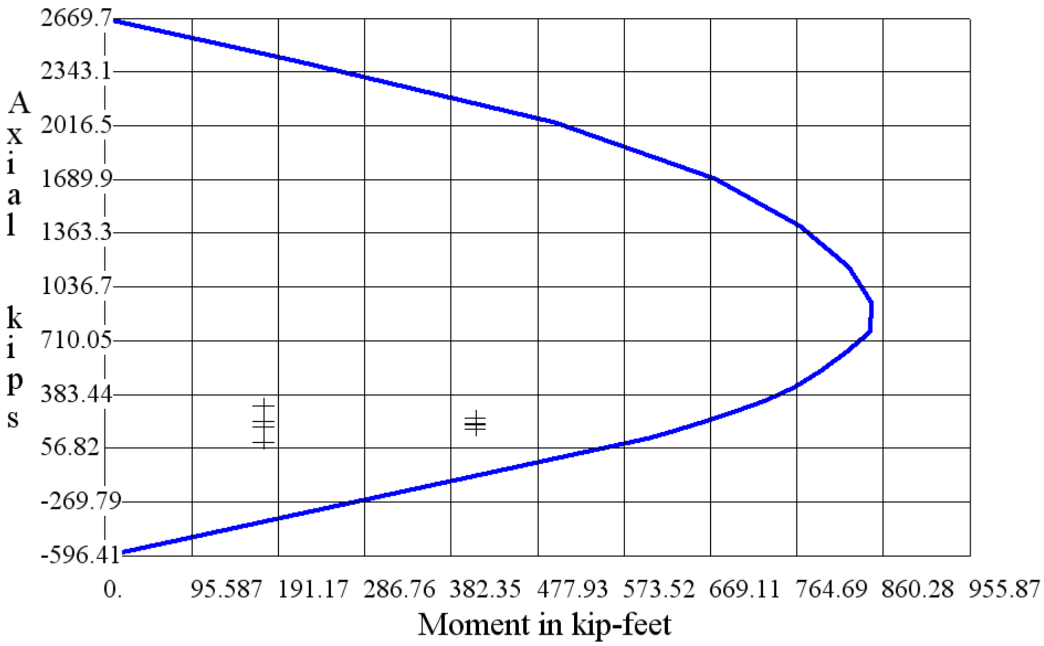
$$P^D/\phi = 70.1/0.75 = 93.5 \text{ kips}$$

$$P^D/\phi = 235.9/0.75 = 314.5 \text{ kips}$$

For design of the columns, try:

10 - #9 bars (Gr. 60 A706 Bars) with #5 Spiral and a Clear Cover of 2 in. The center-to-center spacing of the vertical steel is about 7 ½ in. which is less than the suggested 8 in. maximum. The percentage of steel in relation to the gross area of the column is about 1.4% which is well within the realm of reasonable for this bridge and the accelerations associated with the 500 year design earthquake.

The column interaction diagram is shown below. The load cases calculated above are superimposed on the diagram along with the load cases for the middle columns.



1000 Year Return Period Column Design:

Since the ϕ factor is one for this case, the design axial forces and moments calculated above do not need to be transformed in order compare them to a "nominal" or unreduced column interaction diagram. The forces from Load Cases 1 and 2 are repeated below.

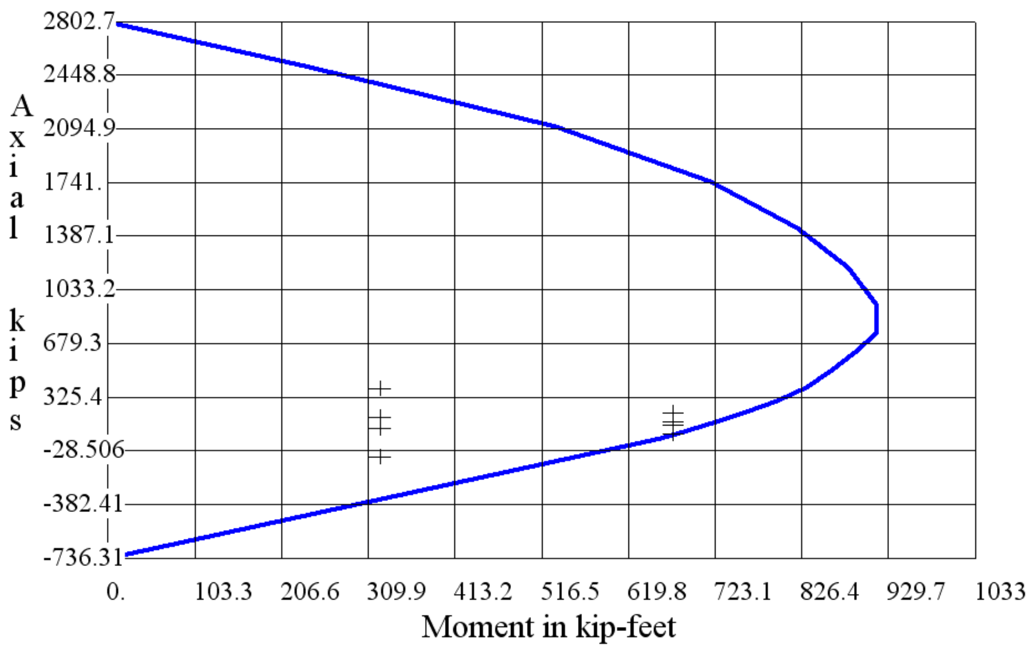
Load Case 1 – Longitudinal Dominant	Load Case 2 – Transverse Dominant
$M^D = 673.6 \text{ k-ft.}$	$M^D = 324.7 \text{ k-ft.}$
$P^D = 85.6 \text{ kips}$	$P^D = -71.7 \text{ kips}$
$P^D = 220.4 \text{ kips}$	$P^D = 377.7 \text{ kips}$

For design of the columns, try:

10 - #10 bars (Gr. 60 A706 Bars) with #5 Spiral and a Clear Cover of 2 in. The center-to-center spacing of the vertical steel is about 7 ½ in. which is less than the suggested 8 in. maximum. The percentage of steel in relation to the gross area of the column is about 1.8%

which is well within the realm of reasonable for this bridge and the accelerations associated with the 1000 year design earthquake.

The column interaction diagram is shown below. The load cases calculated above are superimposed on the diagram along with the load cases for the middle columns.



7.b. Column Design for Shear

For multiple column bents, the Department prefers that spirals and ties have a constant spacing for the full length of the column and required extensions into the cap beam and crashwalls. Various detailing options for shear reinforcement are permitted and described in Section 3.15.5 of the Bridge Manual. Specific details are not covered in this forum. Rather, methods for determining the required bar sizes and spacing for shear reinforcement which satisfy the LRFD and LFD Specifications are focused on.

The design requirements for shear are similar for Category B (LFD) and Zone 3 (LRFD). A minimum amount of steel is required in plastic hinging regions for “confinement” to ensure flexural capacity integrity during an earthquake and the spirals or ties shall also satisfy strength requirements. In Category B, the elastic shear (calculated above for this example) should be used for design. For Zone 3, however, the lesser of the elastic shear or the shear which causes plastic hinging in the columns may be used for design. The shear which causes plastic hinging can be determined from a simple axial-flexural "overstrength" capacity analysis. As recommended in Section 3.15.5.1 of the Bridge Manual, it is simplest to assume that the concrete strength is zero (V_c = 0.0) when designing columnar shear reinforcement without verifying whether some nominal value for V_c is allowed to be considered by LRFD or LFD. Typically, at least the outer columns in multiple column bents either have a low compressive design force or are in tension. When columns are in tension, the shear strength of the concrete should be taken as zero according to LRFD and LFD. The following are equations which should be used for the design of spirals in round columns with $V_c = 0$.

Minimum steel required for confinement expressed as a volumetric ratio:

$$\rho_s \geq 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f'_c}{f_{yh}}$$ (LRFD Eq. 5.7.4.6-1, LFD Div. I-A Eq. 6-4 and 7-4)

And,

$$\rho_s \geq 0.12\frac{f'_c}{f_{yh}}$$ (LRFD Eq. 5.10.11.4.1d-1, LFD Div. I-A Eq. 6-5 and 7-5)

Where:

A_g = gross area of concrete section (in.^{2})

A_c = area of core measured to the outside diameter of the spiral (in.^{2})

f'_c = compressive strength of concrete (ksi)

f_{yh} = yield strength of spiral reinforcement (ksi)

Strength of provided steel:

$$\phi V_s = \phi\frac{A_v f_{yh} d_v}{s}$$ (LRFD Eq. 5.8.3.3-4, LFD Eq. 8-53)

Where:

A_v	=	area of shear reinforcement within a distance s (in.^{2})
d_v	=	effective shear depth (in.)
s	=	spacing of spiral (in.)
ϕ	=	0.9 for LRFD and 0.85 for LFD (however, use 0.9 for LFD)

The equation above for LRFD has been simplified according to the provisions in Article 5.8.3.4.1.

The equations for minimum required confinement steel for tied rectangular or trapezoidal columns are similar to those for round columns and found in the same referenced sections above. When cross ties are used, which is common, they are counted in the total reinforcement area resisting the design shear. The shear requirements for wall type piers are somewhat different than for columns. However, they are not complex. If a wall is designed as a column in its weak direction, the shear design method is the same as that for a column. If a wall is not designed as column in its weak direction, it shall be designed for shear in the same manner as for the strong direction. The minimum reinforcement ratios and shear strength design equations for walls (in the strong and possibly the weak direction) are given in LRFD 5.10.11.4.2 and LFD Div. I-A 7.6.3. The provisions in both specifications are comparable.

The shear design calculations for the 500 and 1000 year design return period earthquakes for the Example 1 bridge are given below using the elastic design forces. The method for calculating the plastic design shear using "overstrength" is demonstrated afterward for the 1000 year earthquake case. Note, however, that for many typical bridges in Illinois, even those in Zones 3 and 4, the shear requirements might easily be met by using the elastic design shears.

500 Year Return Period Shear Design:

Minimum steel required,

$$\rho_s \geq 0.45\left(\frac{A_g}{A_c}-1\right)\frac{f'_c}{f_{yh}} = 0.45\left(\frac{\pi\times15^2}{\pi\times13^2}-1\right)\frac{3.5}{60} = 0.0087$$

And,

$$\rho_s \geq 0.12\frac{f'_c}{f_{yh}} = 0.12\frac{3.5}{60} = 0.0070$$

Try #5 spirals at a spacing of 4 in. center-to-center,

$$\rho_s = \frac{\text{Volume of 1 spiral turn}}{\text{Volume of concrete in 1 spiral turn}} = \frac{4\left(A_v/2\right)}{D_{core} \times s} = \frac{4 \times 0.31}{(30-2-2)\times 4} = 0.0119 > 0.0087 \text{ OK}$$

D_{core} is the diameter of the column out-to-out of the spiral and A_v is the area of 2 bars.

$$\phi V_s = \phi\frac{A_v f_{yh} d_v}{s} = 0.9\frac{2 \times 0.31 \times 60 \times 20.27}{4} = 169.7 \text{ kips}$$

The effective depth, d_v, may be calculated with the method suggested below in the commentary of the LRFD Specifications.

d_v = $0.9d_e = 0.9\left(\frac{D}{2}+\frac{D_r}{\pi}\right) = 0.9\left(\frac{30}{2}+\frac{23.62}{\pi}\right) = 20.27 \text{ in.}$ (LRFD C5.8.2.9)

Where:

D = gross diameter of column (in.)

D_r = diameter of the circle passing through the centers of longitudinal reinforcement (in.)

Comparing the elastic design shear forces for Load Case 1 and Load Case 2 gives:

Load Case 1: 63.6 < 169.7 OK

Load Case 2: 53.5 < 169.7 OK

$\therefore$#5 spiral at 4 in. center-to-center spacing OK (and the comparisons above indicate that a spacing as large as 6 in. center-to-center may also be acceptable).

1000 Year Return Period Shear Design:

Minimum steel required,

$$\rho_s \geq 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f_c'}{f_{yh}} = 0.0087 \text{ and } \rho_s \geq 0.12\frac{f_c'}{f_{yh}} = 0.0070$$

Try #5 spirals at a spacing of 4 in. center-to-center,

$$\rho_s = \frac{4\left(A_v/2\right)}{D_{core} \times s} = 0.0119 > 0.0087 \text{ OK}$$

$$\phi V_s = \phi\frac{A_v f_{yh} d_v}{s} = 0.9\frac{2 \times 0.31 \times 60 \times 20.23}{4} = 169.3 \text{ kips}$$

$$d_v = 0.9d_e = 0.9\left(\frac{D}{2} + \frac{D_r}{\pi}\right) = 0.9\left(\frac{30}{2} + \frac{23.48}{\pi}\right) = 20.23 \text{ in. (LRFD C5.8.2.9)}$$

Comparing the elastic design shear forces for Load Case 1 and Load Case 2 gives:

Load Case 1: 141.2 < 169.3 OK
Load Case 2: 142.0 < 169.3 OK

$\therefore$#5 spiral at 4 in. center-to-center spacing OK.

7.c. 1000 Year Return Period Plastic Shear Determination Using Overstrength

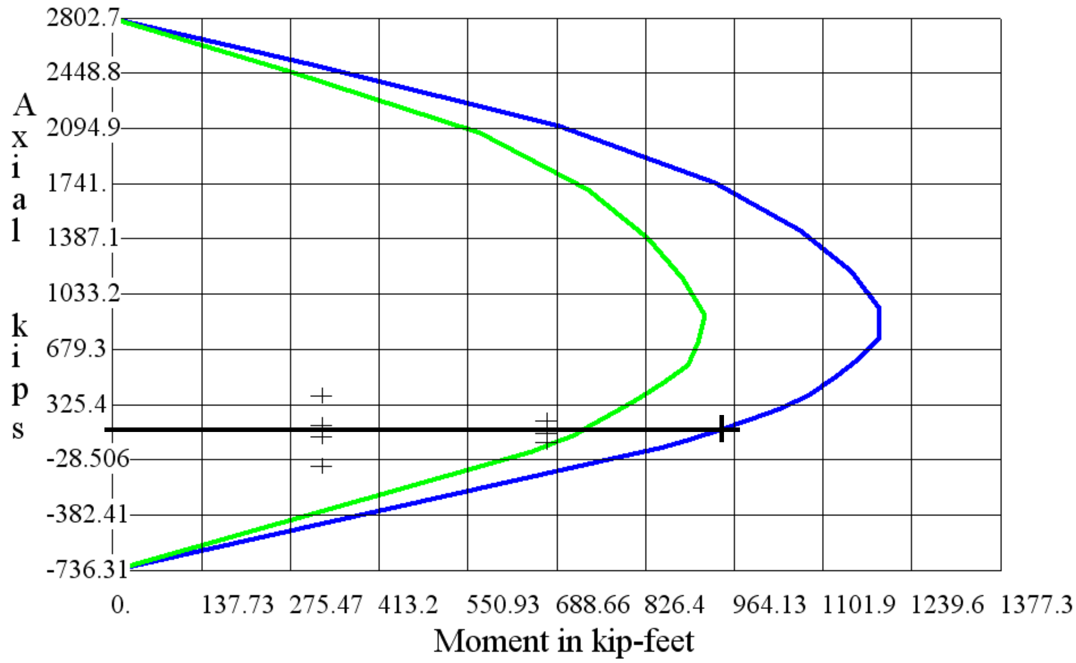
In Illinois, the determination of plastic shear capacity using overstrength should generally be confined to bridge types for which the plastic hinges in the substructure elements would form above ground at the piers during the design earthquake. This typically entails bents with cap beams, crashwalls and multiple columns which are either circular or trapezoidal.

However, the Department does not discourage overstrength analysis for such pier types as individual column drilled shaft bents or drilled shaft bents with web walls.

Once the columns have been designed to resist axial forces and moments in a ductile manner (with an R-Factor), the other components of the bridge can be designed for the lesser of the base shear which actually causes the columns to form plastic hinges or the elastic forces from the original analysis. The “overstrength” of a column can be thought of as a simplified engineering estimate of the base shear required to cause plastic hinging. For typical cases in Illinois, these components are usually only the columnar shear reinforcement and the piles.

Overstrength analysis is reserved for bridges located in regions where design accelerations are considered significant. For the 500 year design return period earthquake, this translates to LFD Seismic Performance Categories C and D and for the 1000 year seismic event it corresponds to LRFD Seismic Performance Zones 3 and 4 (and 2, but not explicitly).

The overstrength column capacity should be calculated by one of two methods depending on the levels of axial forces used to design the vertical steel in a column. When the design axial forces generally fall below the balanced failure point on the nominal column axial force-moment interaction diagram, the overstrength of a concrete column should be determined by only multiplying the moment strength by 1.3. If the design axial forces generally fall above the balanced failure point, the nominal axial and moment strengths should both be multiplied by 1.3. Once the overstrength curve has been calculated, a simple procedure is used to determine the plastic shear capacities for the longitudinal and transverse directions. This procedure is outlined in LRFD Article 3.10.9.4.3 and LFD Div. I-A Article 7.2.2 for piers with either single or multiple columns. A plot of the overstrength capacity for the pier columns of the Example 1 bridge is shown below for the 1000 year design earthquake.



Since the axial design forces fall below the balance point, only the nominal moment strengths were multiplied by 1.3. The calculations and descriptions below detail the method for determining the plastic shear capacities for the transverse and longitudinal directions of the Example 1 bridge.

Initial Plastic Moment Capacity:

For multiple column bents, the initial design axial force used to determine the initial corresponding plastic moment capacity should be that from the dead load only. Referring to the overstrength axial force-moment interaction diagram above,

Initial Axial Dead Load	=	153 kips
Initial Plastic Moment	=	944 kip-ft.

Initial Plastic Shear Capacity:

The plastic shear capacities for the transverse and longitudinal directions are found with the same basic statics equations used above for determining elastic column moments. The unknowns to solve for, though, are shears instead of moments.

Long. Shear

$$M_P = 944 \text{ kip-ft.} = V_{\text{Plastic Long.}} \times (12.5 \text{ ft.} + 4 \text{ ft.})$$
$$\therefore V_{\text{Plastic Long.}} = 944/16.5 = 57.2 \text{ kips/column}$$
$$\text{and } V_{\text{Plastic Long. Bent}} = 4 \times 57.2 = 228.8 \text{ kips}$$

Trans. Shear

$$M_P = 944 \text{ kip-ft.} = V_{\text{Plastic Trans.}} \times \left(\frac{12.5 \text{ ft.}}{2}\right)$$
$$\therefore V_{\text{PlasticTrans.}} = 944 \times 2/12.5 = 151.0 \text{ kips/column}$$
$$\text{and } V_{\text{Plastic Trans. Bent}} = 4 \times 151 = 604.0 \text{ kips}$$

Initial Overturning and Frame Action Axial Forces from Initial Plastic Base Shears:

Longitudinal — There are no overturning or frame action axial forces from base shears applied in the longitudinal direction. Consequently, the initial longitudinal plastic shear calculated above is the final plastic shear.

Transverse — Using methods from the elastic analyses above.

Overturning:

S_p (Base Shear at Pier) = 604.0 kips

M (Overturning Moment) = $604 \times 7.8125 = 4718.8 \text{ k-ft.}$

P_{ST} (Max. Axial Col. Force) = $\frac{3}{10} 4718.8/12 = 118.0 \text{ kips}$

Frame Action:

S_p (Base Shear at Pier) = 604.0 kips

V_{ST} (Shear Per Column) = $604.0/4 = 151.0 \text{ kips}$

M_{ST} (Moment Per Column) = $151.0 \times 12.5/2 = 943.8$ k-ft.

P_{SB} (Max. Axial Col. Force) = $1.67 \times 943.8/12 = 131.3$ kips

Total "Plastic" Axial Force:

$P^{DPlastic}$ = 153.0 + 118.0 + 131.3 = 402.3 kips

Revised Overstrength Moment Resistance:

Long. Not required (see above).

Trans. Taking $P^{DPlastic}$ (402.3 kips) as the design axial force (instead of just the dead load) and referring again to the overstrength axial force-moment interaction diagram above, a plastic moment of 1082 kip-ft. is obtained. The corresponding plastic shear per column is 173.1 kips and for the bent the plastic shear is about 692.4 kips. This plastic base shear is within about 15% of the first base shear value (604.0 kips). According to the method, a further iteration should be performed such that successive values of calculated plastic base shears are within 10% of each other. The next iteration would start at the "Overturning and Frame Action Axial Forces from Plastic Base Shears" step with a base shear of 692.4 kips. However, another iteration is not necessary because the calculated plastic base shear has already been shown to be greater than the design elastic shear (544.2 kips). As such, the elastic force may be used for design.

Overstrength Summary:

Longitudinal	Elastic Base Shear	=	540.7 kips
	Plastic Base Shear	=	228.8 kips
Transverse	Elastic Base Shear	=	544.2 kips
	Plastic Base Shear	=	≈ 692 kips

According to the provisions of LFD Div. I-A, it can be interpreted as acceptable to use the plastic longitudinal base shear in conjunction with the elastic transverse base shear for the design of the columnar shear reinforcement and the piles for the Example 1 bridge. This may not be the case for LRFD, though. LRFD appears to steer the engineer towards "consistently" using either the elastic or plastic cases. Either the LFD or apparent LRFD approach, however, is permitted by the Department (regardless of which code the bridge is being designed under) and left to the designer's judgment.

The results of the overstrength analysis and their potential impact on the design of the Example 1 bridge are somewhat typical for multiple column bents in Illinois. As shown above, the benefits of overstrength analysis were not required to design the columnar shear reinforcement with a reasonable spiral size and spacing for the 1000 yr. seismic event. As such, the use of overstrength analysis should be carefully considered by the designer. For many cases in Illinois, it is more straightforward and simpler to use the elastic design forces calculated from the initial determination of the bridge periods.

8. Pile Design Overview

Seismic design forces for the piles at piers and abutments in the Example 1 bridge can be determined with methods which are analogous to those described above for the pier columns in the transverse direction. Each row of piles at the piers along the transverse and longitudinal directions can be treated as an individual frame in the soil as if it were a column bent. For example, consider the case where the pier foundations for the Example 1 Bridge has two rows of ten piles. In the transverse direction, there would be two pile bent frames with 9 "portals", each analyzed as taking half the transverse base shear at a pier. In the longitudinal direction, there would be ten single portal frames with each analyzed as taking one-tenth of the longitudinal base shear. The overturning "moment arm" is longer for the piles and should extend down to the bottom of the footing. Note that there is overturning moment for both the transverse and longitudinal directions. In the longitudinal direction, the analysis is not for a cantilever as it is for the columns. It is most straightforward to use the elastic base shears in the transverse and longitudinal directions in which the design moments are unreduced (R = 1.0).

For many cases, the pile fixity depths for steel H-piles provided in Appendix C (which are presented in terms of "moment fixity" and not "deflection fixity") can be used for design.

Guidance on the appropriateness of their use is also provided in Appendix C. Overturning analysis solutions for cases with up to 13 columns or piles in a frame are provided in Appendix A. Frame analysis solutions for any number of portals are provided in Appendix B. Example 4 may also be referenced for additional guidance concerning simple structural analysis techniques for piles in global and local models. More complex or sophisticated analysis methods are not discouraged by the Department and may be appropriate depending on the bridge being designed.

"Group Action" for piles may be considered by the designer, but this is not required when using the methods presented in the examples and appendices of this design guide for regular or typical bridges.

Structurally, the piles should be designed for shear, and combined axial force (including compression and tension as appropriate) and bending. Piles can be assumed to be continually braced throughout their length in the soil. Pullout of the piles from the footing is also a consideration. When required, special seismic pile anchorage details in footings should be used and are provided in Section 3.15.5.5 of the Bridge Manual. Reference Parts 8 and 9 of Example 4 for example calculations that check the structural capacity of steel H-piles.

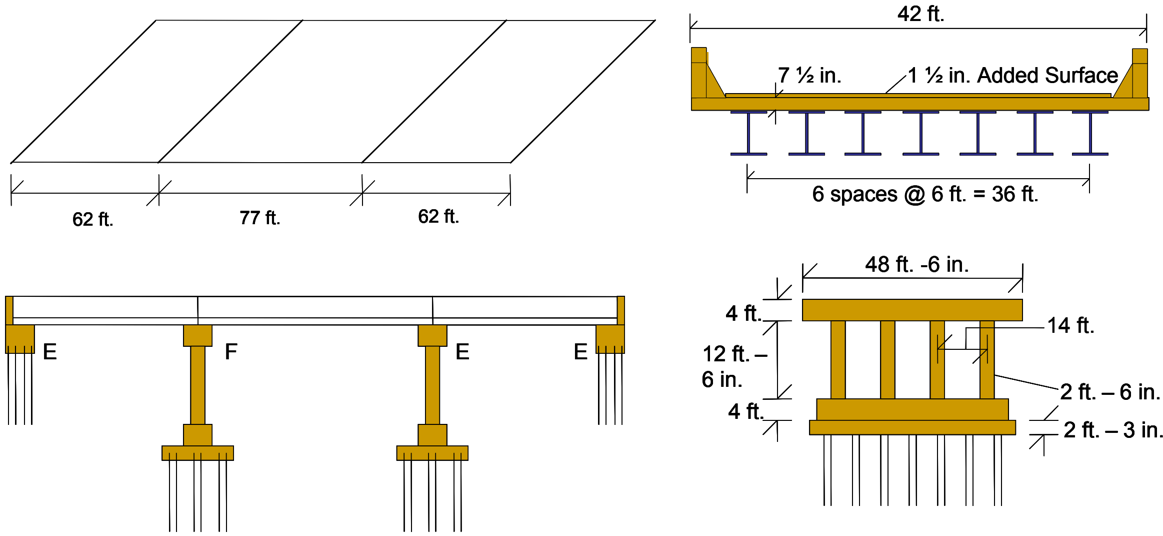
The piles in the Example 1 bridge could also be metal shell. Metal shell piles should be designed structurally for shear, and combined axial force and moment just as H-piles are. Metal shell piles behave as reinforced concrete columns in soil with the shell acting as the reinforcement. Supplemental longitudinal (vertical) and shear/confinement (spiral) reinforcement may also be provided inside of metal shell piles to increase structural capacity. Appendix C provides fixity depths, and nominal axial force–moment strength interaction diagrams for metal shell piles without supplemental reinforcement. See Section 3.15.5.5 of the Bridge Manual and Part 14 of Example 4 for additional guidance.

Geotechnical considerations for the design of piles to resist seismic loadings are provided in Sections 3.10 and 3.15 of the Bridge Manual as well as Design Guide 3.10.1. Guidance on liquefaction, lateral pile resistance in soil, pullout from the soil, etc. are among the considerations addressed.

Example 2

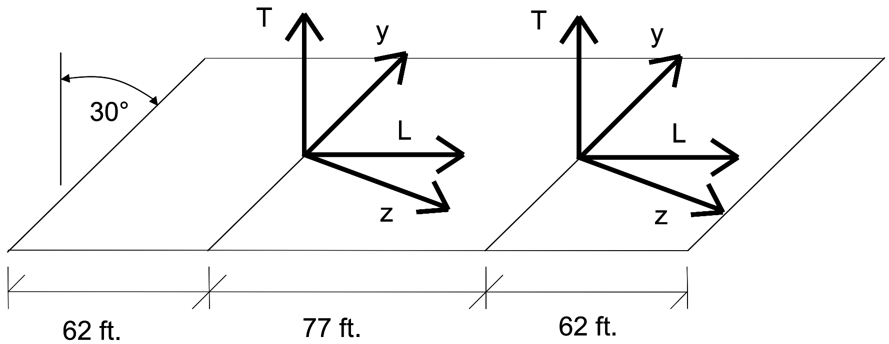
Example 1 Bridge with a Skew of 30° for the 500 Year Design Earthquake Return Period

The primary focus of this example is to examine the differences from Example 1 in how pier column seismic design forces are calculated when a bridge is skewed. The methods described in Example 2 are also applicable to pile seismic design force calculations for this and other similar bridge types which are skewed. The secondary focus of Example 2 is the effects of skew on the determination of bridge periods and base shears in the transverse and longitudinal directions.



1. Determination of Bridge Periods and Base Shears – 500 Year Design Earthquake Return Period

For Example 2, assume that the weight of the bridge used for seismic/dynamic analysis calculations is the same as that for Example 1 (there is only a nominal increase in weight due to the increased length of the cap beams). Further assume that the stiffness of the abutments is the same as that for the bridge in Example 1 even though the piles are at a 30° skew. In reality, the abutment stiffnesses are comparable to those for the Example 1 case (See methods of Example 3 Part 2 and in Appendix C). Consequently, this assumption does not introduce a significant "error" in the dynamic analysis of the structure. The "global" transverse and longitudinal axes of the bridge are shown below along with the "local" pier axes.



For design, the transverse and longitudinal stiffnesses of the piers can be taken as the same as the Example 1 bridge with no skew. This is because the columns are round and their stiffnesses were based only on the columns deforming in reverse curvature (transverse) and as cantilevers (longitudinal). If the columns were of rectangular or trapezoidal cross-section, this assumption would not be valid. Example 3 considers the more complex cases of a skewed bridge with rectangular and trapezoidal columns.

Based upon the discussion above, it is acceptable for design purposes to take the transverse and longitudinal periods of the bridge as the same for both the skewed and non-skewed cases (Example 2 and Example 1, respectively). Even if the stiffnesses of the abutment piles were modeled more "correctly", the differences in the periods and calculated base shears would not be considered to have engineering significance. Since the periods are essentially equivalent for the Example 1 and 2 structures, the base shears are also equivalent. These are summarized again from Example 1 for the 500 yr. seismic design event.

Transverse Period	=	0.18 sec.
Pier Base Shear ($V_{Base\ Shear\ P(T)}$)	=	200.9 kips
Abut. Base Shear ($V_{Base\ Shear\ A\ (T)}$)	=	69.4 kips
Longitudinal Period	=	0.50 sec.
Pier Base Shear ($V_{Base\ Shear\ P(L)}$)	=	247.2 kips
Abut. Base Shear ($V_{Base\ Shear\ A\ (L)}$)	=	0 kips

2. Frame Analysis and Columnar Seismic Forces for Multiple Column Bent – 500 Year Design Earthquake Return Period

When a bridge is skewed, the global transverse and longitudinal pier and abutment axes do not coincide with the local axes as shown above. As such, a separate local transverse (y-axis) and longitudinal (z-axis) axis force analysis is required for both the global transverse (T-axis) and longitudinal (L-axis) axes. These analyses are presented below for the exterior columns. Note that the subscripts and superscripts for the axial forces, shears, and moments presented below are different from Example 1 for this more complex case. They "match" the variables used when the forces are combined orthogonally in Section 3 of this example.

2.a Pier Forces – Dead Load

Design Dead Load Per Column = 153.0 kips (from Example 1)

2.b. Pier Forces from Global Transverse Base Shear

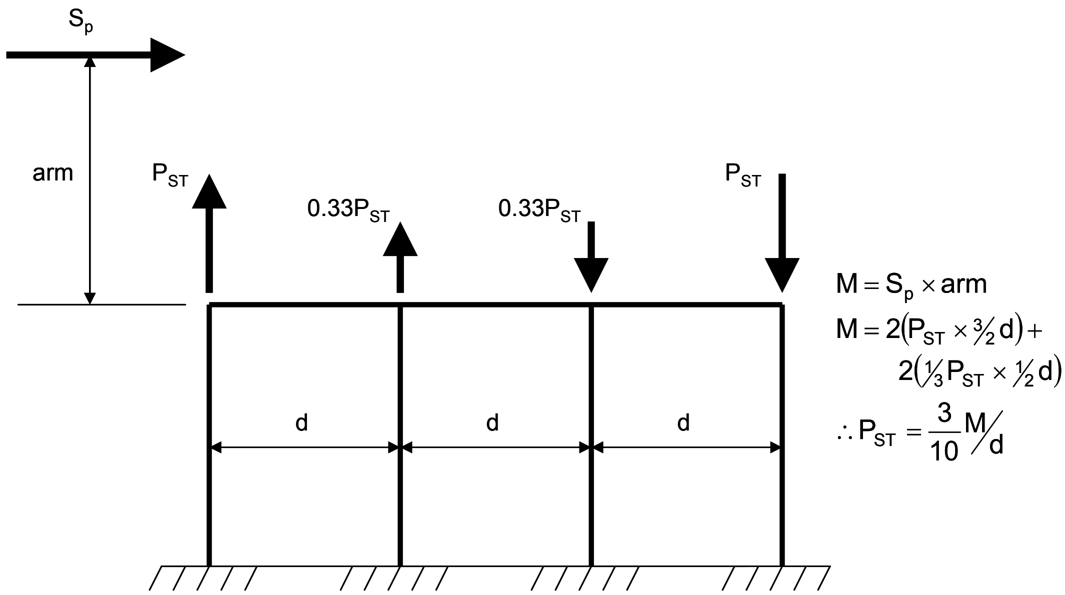
The component of the global transverse (T-axis) base shear in the local transverse (y-axis) direction produces axial forces in the columns from overturning and frame action as well as moments from the columns deforming in reverse curvature. The component of the global transverse base shear in the local longitudinal (z-axis) produces a cantilever moment. The components of the global transverse base shear in the local y- and z-directions are given below.

$V^{T(Bent)} = V_{Base\ Shear\ P(T)} = 200.9\text{ kips}$

$V_y^{T(Bent)} = V^T \cos 30° = 200.9 \cos 30° = 174.0\text{ kips}$

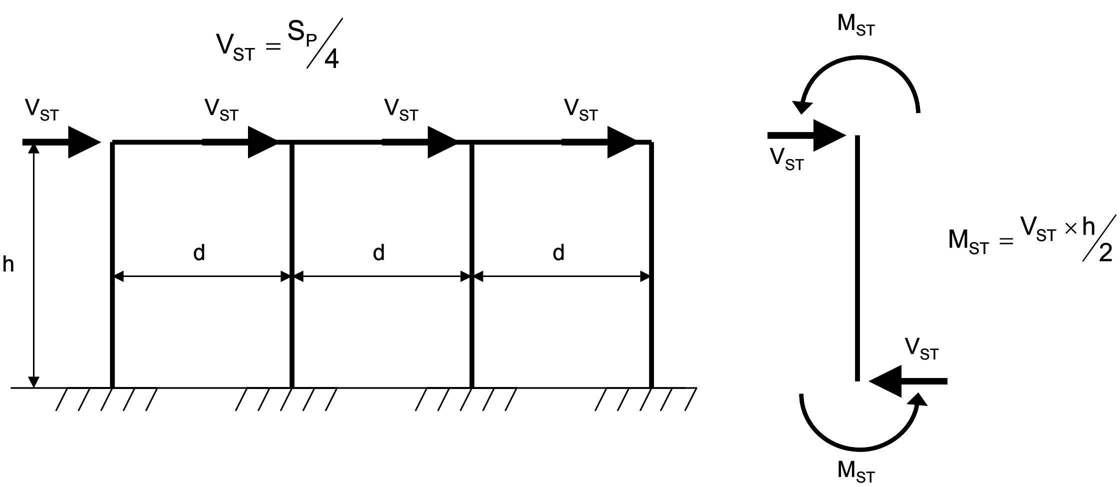
$V_z^{T(Bent)} = V^T \sin 30° = 200.9 \sin 30° = 100.5\text{ kips}$

The forces from overturning (moments about local z-axis, forces in local y-axis direction) are,



$V_y^{T(Bent)}$ =	S_p	=	174.0 kips
arm		=	7.8125 ft. (Example 1)
d		=	14 ft.
M (Overturning Moment)		=	$174.0 \times 7.8125 = 1359.4$ k - ft.
P_S^T =	P_{ST}	=	$\frac{3}{10} 1359.4/14 = 29.1$ kips

The forces from frame action (moments about local z-axis, forces in local y-axis direction) are,

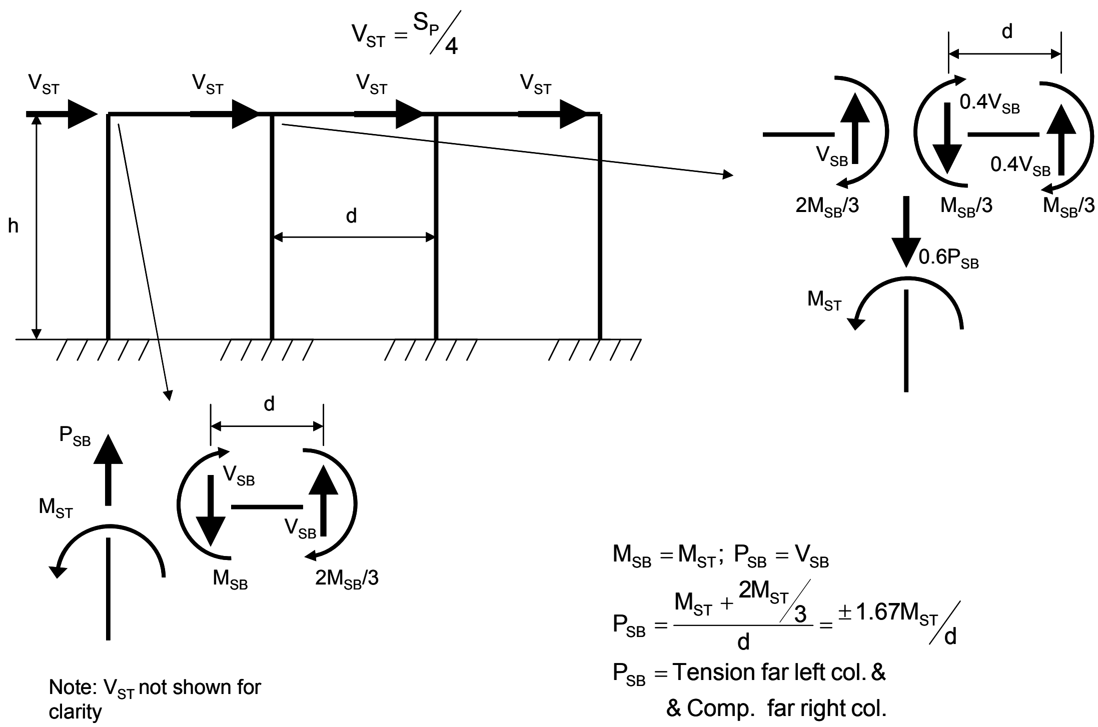


$V_y^{T(Bent)}$ = S_p = 174.0 kips

Column Height (Clear) = 12.5 ft.

V_y^T = V_{ST} = $174.0/4 = 43.5$ kips

M_z^T = M_{ST} = $43.5 \times 12.5/2 = 271.9$ k - ft.



M_z^T = M_{ST} = 271.9 k-ft.

d = 14 ft.

P_B^T = P_{SB} = $1.67 \times 271.9/14 = 32.4$ kips

The moments about the local y-axis, and forces in the local z-axis direction are,

S_L = 100.5 kips

Column Height (Clear) = 12.5 ft.

Cap Beam Height = 4.0 ft.

V_z^T = V_{SL} = $100.5/4 = 25.1$ kips

M_y^T = $M_{ColBot(SLB)}$ = $25.1 \times (12.5 + 4) = 414.2$ k-ft.

2.c. Pier Forces from Global Longitudinal Base Shear

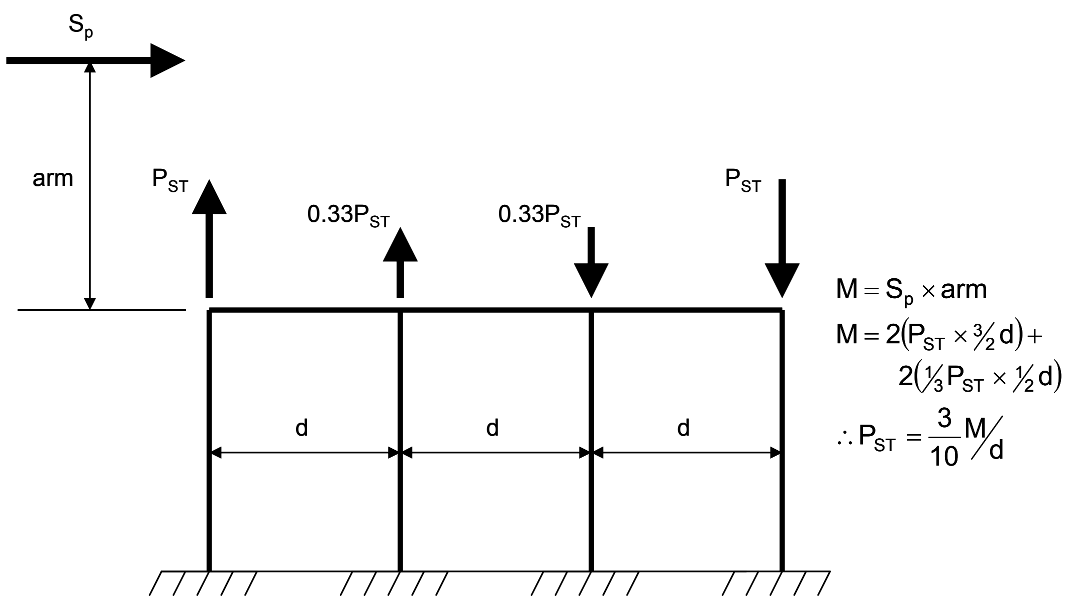
The component of the global longitudinal (L-axis) base shear in the local transverse (y-axis) direction produces axial forces in the columns from overturning and frame action as well as moments from the columns deforming in reverse curvature. The component of the global longitudinal base shear in the local longitudinal (z-axis) produces a cantilever moment. The components of the global longitudinal base shear in the local y- and z-directions are given below.

$V^{L(Bent)}$ = $V_{Base\ Shear\ P(T)}$ = 247.2 kips

$V_y^{L(Bent)}$ = $V^L \sin 30°$ = $247.2 \sin 30°$ = 123.6 kips

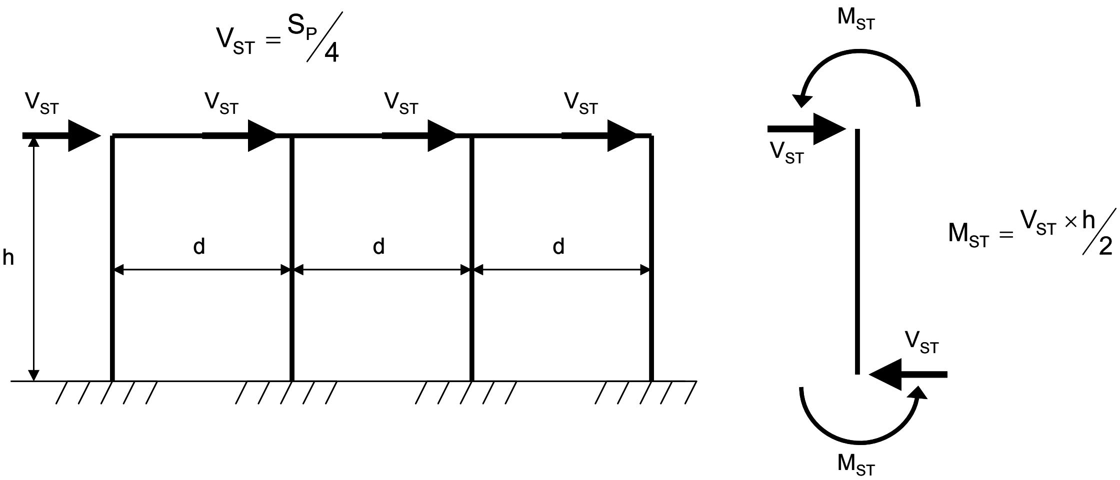
$V_z^{L(Bent)}$ = $V^L \cos 30°$ = $247.2 \cos 30°$ = 214.1 kips

The forces from overturning (moments about local z-axis, forces in local y-axis direction) are,

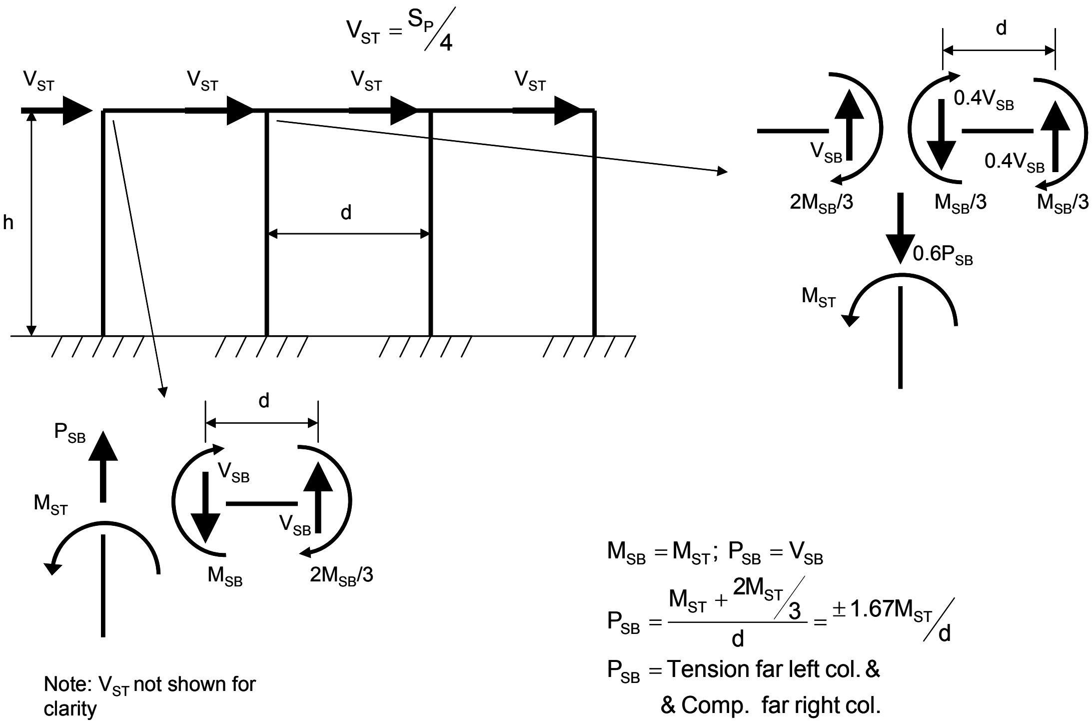


$V_y^{L(Bent)}$	=	S_p	=	123.6 kips
arm			=	7.8125 ft. (Example 1)
d			=	14 ft.
M (Overturning Moment)			=	$123.6 \times 7.8125 = 965.6$ k - ft.
P_S^L	=	P_{ST}	=	$\frac{3}{10} 965.6 / 14 = 20.7$ kips

The forces from frame action (moments about local z-axis, forces in local y-axis direction) are,



$V_y^{L(Bent)}$	=	S_p	=	123.6 kips
Column Height (Clear)			=	12.5 ft.
V_y^L	=	V_{ST}	=	$123.6 / 4 = 30.9$ kips
M_z^L	=	M_{ST}	=	$30.9 \times 12.5 / 2 = 193.1$ k - ft.



M_z^L	=	M_{ST}	=	193.1 k-ft.
d			=	14 ft.
P_B^L	=	P_{SB}	=	$1.67 \times 193.1/14 = 23.0$ kips

The moments about the local y-axis, and forces in the local z-axis direction are,

S_L			=	214.1 kips
Column Height (Clear)			=	12.5 ft.
Cap Beam Height			=	4.0 ft.
V_z^L	=	V_{SL}	=	$214.1/4 = 53.5$ kips
M_y^L	=	$M_{ColBot(SLB)}$	=	$53.5 \times (12.5 + 4) = 882.8$ k - ft.

3. *Seismic Design Forces for Multiple Column Bent Including R-Factor, P-Δ, and Combination of Orthogonal Forces – 500 Year Design Earthquake Return Period*

3.a. R-Factor

R-Factor = 3.5 (from Example 1)

3.b. P-Δ

P-Δ Amplification = 1.05 (from Example 1)

3.c. Summary and Combination of Orthogonal Column Forces Used for Design

The equations for combination of orthogonal forces are repeated below from Example 1. Because the bridge is skewed, none of the terms will be equal to zero for Example 2. R-Factors and P-Δ amplification effects are also included in the calculations.

Load Case 1 (Longitudinal Dominant)

$$V_z^D = 1.0\left|V_z^L\right| + 0.3\left|V_z^T\right|$$

$$V_y^D = 1.0\left|V_y^L\right| + 0.3\left|V_y^T\right|$$

$$M_z^D = 1.0\left|M_z^L\right| + 0.3\left|M_z^T\right|$$

$$M_y^D = 1.0\left|M_y^L\right| + 0.3\left|M_y^T\right|$$

$$P^D = 1.0\left|P^L\right| + 0.3\left|P^T\right|$$

Load Case 2 (Transverse Dominant)

$$V_z^D = 0.3\left|V_z^L\right| + 1.0\left|V_z^T\right|$$

$$V_y^D = 0.3\left|V_y^L\right| + 1.0\left|V_y^T\right|$$

$$M_z^D = 0.3\left|M_z^L\right| + 1.0\left|M_z^T\right|$$

$$M_y^D = 0.3\left|M_y^L\right| + 1.0\left|M_y^T\right|$$

$$P^D = 0.3\left|P^L\right| + 1.0\left|P^T\right|$$

Load Case 1 – Longitudinal Dominant:

$$V_z^D = 1.0\left|V_z^L\right| + 0.3\left|V_z^T\right| = 1.0\left|53.5\right| + 0.3\left|25.1\right| = 61.0 \text{ kips}$$

$$V_y^D = 1.0\left|V_y^L\right| + 0.3\left|V_y^T\right| = 1.0\left|30.9\right| + 0.3\left|43.5\right| = 44.0 \text{ kips}$$

$$M_z^D = 1.0\left|M_z^L\right| + 0.3\left|M_z^T\right| = 1.0\left|1.05 \times 193.1/3.5\right| + 0.3\left|1.05 \times 271.9/3.5\right| = 82.4 \text{ k-ft.}$$

$$M_y^D = 1.0\left|M_y^L\right| + 0.3\left|M_y^T\right| = 1.0\left|\frac{1.05 \times 882.8}{3.5}\right| + 0.3\left|\frac{1.05 \times 414.2}{3.5}\right| = 302.1 \text{ k-ft.}$$

$$P^D = 1.0\left|P^L\right| + 0.3\left|P^T\right| \rightarrow P^D = 153.0 \pm 1.0\left|20.7 + 23.0\right| \pm 0.3\left|29.1 + 32.4\right| = 90.9 \text{ and } 215.2 \text{ kips}$$

(Note: Axial Dead Load = 153 kips)

The design shears and moments can be added (as vectors) since the columns are round in order to further simplify the design forces.

$$V^D = \sqrt{61.0^2 + 44.0^2} = 75.2 \text{ kips}$$

$$M^D = \sqrt{82.4^2 + 302.1^2} = 313.1 \text{ k-ft.}$$

$$P^D = 90.9 \text{ and } 215.2 \text{ kips}$$

Load Case 2 – Transverse Dominant:

$$V_z^D = 0.3\left|V_z^L\right| + 1.0\left|V_z^T\right| = 0.3\left|53.5\right| + 1.0\left|25.1\right| = 41.2 \text{ kips}$$

$$V_y^D = 0.3\left|V_y^L\right| + 1.0\left|V_y^T\right| = 0.3\left|30.9\right| + 1.0\left|43.5\right| = 52.8 \text{ kips}$$

$$M_z^D = 0.3\left|M_z^L\right| + 1.0\left|M_z^T\right| = 0.3\left|\frac{1.05 \times 193.1}{3.5}\right| + 1.0\left|\frac{1.05 \times 271.9}{3.5}\right| = 98.9 \text{ k-ft.}$$

$$M_y^D = 0.3\left|M_y^L\right| + 1.0\left|M_y^T\right| = 0.3\left|\frac{1.05 \times 882.8}{3.5}\right| + 1.0\left|\frac{1.05 \times 414.2}{3.5}\right| = 203.7 \text{ k-ft.}$$

$$P^D = 0.3\left|P^L\right| + 1.0\left|P^T\right| \rightarrow P^D = 153.0 \pm 0.3\left|20.7 + 23.0\right| \pm 1.0\left|29.1 + 32.4\right| = 78.4 \text{ and } 227.6 \text{ kips}$$

Further simplification of the design shears and moments leads to the following.

$$V^D = \sqrt{41.2^2 + 52.8^2} = 67.0 \text{ kips}$$

$$M^D = \sqrt{98.9^2 + 203.7^2} = 226.4 \text{ k-ft.}$$

$$P^D = 78.4 \text{ and } 227.6 \text{ kips}$$

Example 3

Overview of Bents with Rectangular or Trapezoidal Columns

Examples 1 and 2, along with their associated discussions provide a number of techniques and guidance which can be used for the seismic bridge design of a large number of cases in Illinois which would be considered typical or common. Example 3 provides an overview of typical situations in Illinois which have not been covered. These are bridges with rectangular or trapezoidal columns, especially those on a skew. The provided discussions can also be extended to the design of bridges with H-piles as appropriate.

1. *Overview of Seismic Design of Multiple Column Bents with Rectangular or Trapezoidal Columns for Bridges with No Skew*

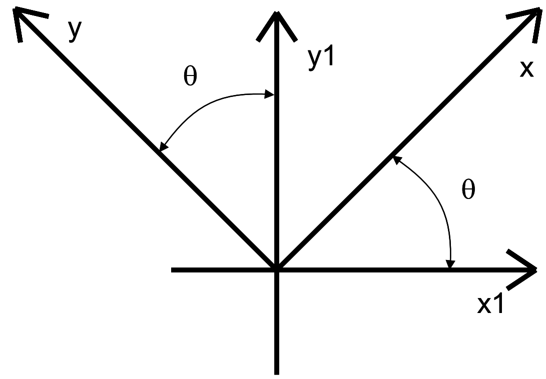
For bridges with rectangular or trapezoidal column bents and no skew, there are only a few differences when designing for seismic loads when compared to designing bridges with round columns. The rectangular column case is the simplest. The techniques from Example 1 can be used with one exception being that there are two column moments of inertia to consider. One is about the longitudinal axis of the bridge (and bent) and the other is about the transverse axis of the bridge (and bent). However, the major complicating difference from bridges with round column bents is the design of the columns for axial force and moment. The moments to consider are bi-axial. At the 500 yr. design level seismic event, more "approximate" methods of bi-axial column design can be used. At the 1000 yr. seismic event level, though, it is recommended that more precise design methods be used, although this is not explicitly required by the Department. It is possible that excessive vertical steel may be called for by not using more sophisticated bi-axial column design techniques which usually entail the use of commercial software.

The seismic design of bridges with no skew and trapezoidal columns adds another level of complexity above that of the consideration of just rectangular columns. Probably the simplest method for developing a global finite element model of the bridge (for the purposes of period and base shear calculations) is to determine equivalent rectangular column moments of inertia for the trapezoidal columns about the transverse and longitudinal axes. The deflections from equivalent rectangular columns can be equalized to those from simple finite element models of a trapezoidal column about the weak and strong axis to obtain two simple equivalent moments of inertia for a column such that a value for the stiffness of the bent can be calculated for both directions. The trapezoidal columns should be “discreetized” into about 5 to 10 finite elements. Note that a hammerhead pier can be considered as having a single trapezoidal column.

At the local model level, when the vertical steel of trapezoidal columns is being designed, a full finite element model of the bent with standard frame elements would probably be the analysis method most designers would choose. However, this is not always necessary, and the method chosen should be based upon the designers experience and judgment. The critical moment location (plastic hinge location) for the design of trapezoidal columns is usually at the smallest cross-sectional slice of the column.

2. Overview of Seismic Design of Multiple Column Bents with Rectangular or Trapezoidal Columns for Bridges with Skew

The techniques from Examples 1 and 2, and the overview above can be used for the design of skewed bridges with rectangular or trapezoidal columns. Presented in this section, though, is one additional method or tool which primarily facilitates the construction of simple global analysis models. The cross-sections of rectangular or trapezoidal columns are “skewed” or rotated when the bridge is skewed. As such, equivalent moments of inertia for rotated rectangular cross-sections can prove very useful for the calculation of bridge periods and base shears. A simple illustration and equations for calculating equivalent moments of inertia for rotated sections are given below.



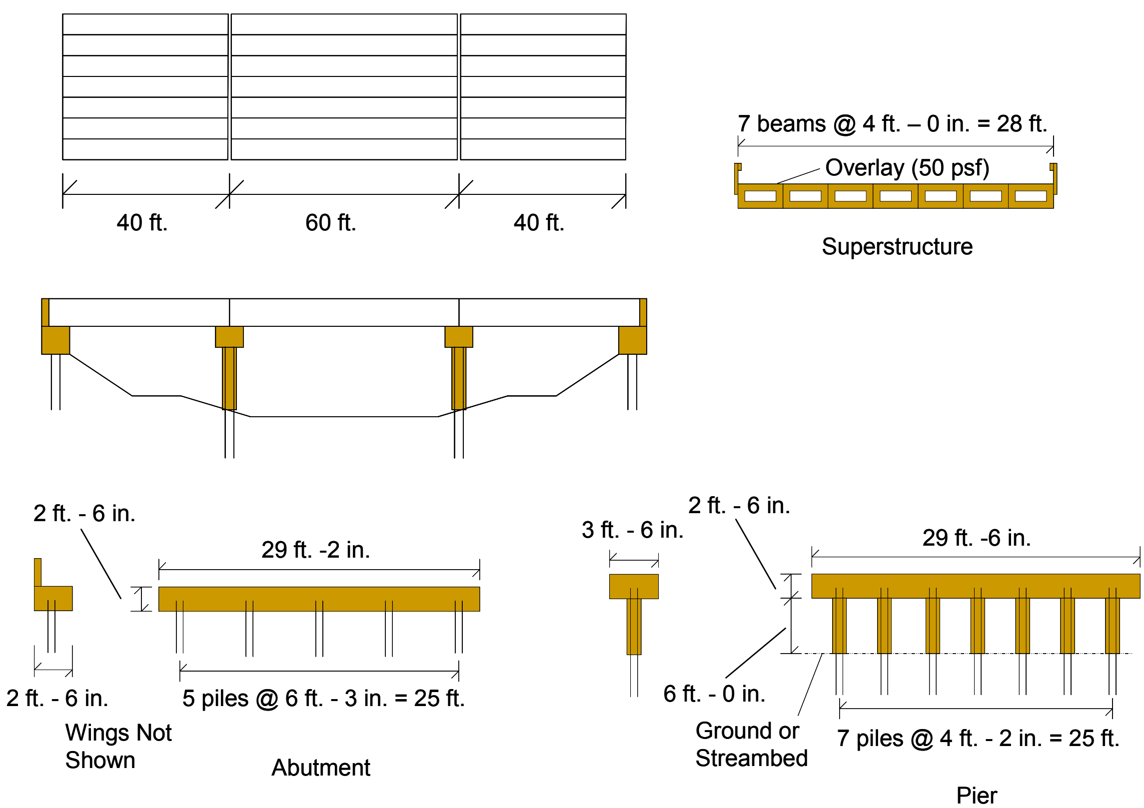
$$I_{x1} = \sqrt{\frac{I_y^2 \times I_x^2}{I_y^2 \cos^2 \theta + I_x^2 \sin^2 \theta}}$$

$$I_{y1} = \sqrt{\frac{I_y^2 \times I_x^2}{I_x^2 \cos^2 \theta + I_y^2 \sin^2 \theta}}$$

Example 4

Design of a Simply Supported Multi-Span PPC Deck Beam Bridge for 1000 yr. Design Return Period Earthquake Using the Flexible Option

The bridge presented in this example is a 3-span structure with stub abutments and pile bent piers. The abutments have a single row of piles and the piles in the piers are individually encased. The beams for each span are simply supported PPC deck beams. The overlay on the deck is assumed to weigh 50 psf and is comprised of either bituminous or non-bituminous concrete. It only adds to the dead weight of the bridge and is non-continuous. The bridge is in a special class compared to the majority of other types of structures designed and built on the State and Local Bridge Systems. However, it is common in Illinois and has been historically designed and constructed mostly on the Local Bridge System.



Beams: IDOT Standard 27 in. x 48 in. PPC Deck Beams

There are several aspects common to the class of bridges represented by the Example 4 structure which makes them special or somewhat unique compared to the bridge types focused on and discussed in Examples 1 to 3. Non-continuity of the superstructure across the piers is probably the most important. Lack of continuity causes the total base shear in the transverse direction to be distributed to the abutments and piers according to simple tributary span lengths as opposed to how it would be distributed using an indeterminate structural analysis required for bridges which have continuous superstructures. The substructures/foundations and superstructures are also generally not as stiff as their counterpart bridge types commonly built in Illinois. Essentially, the superstructures are relatively loosely connected collections of beams which probably do not act together as one when they deflect transversely under seismic loadings and all the substructure/foundation units are only supported by one row of piles. The net effects of less stiff structural elements (particularly at the piers) in combination with superstructure discontinuity include longer transverse periods and fundamental mode shapes which can be somewhat unusual compared to other classes of bridges constructed in Illinois. In addition, the calculation of an estimate of the period is quite simple and reasonably accurate if the stiffnesses of the piers are similar. If pier stiffnesses are overly dissimilar, the estimate of the period and the transverse base shear computed for design will be conservative. This recommended approach, which is described below, is adequate for design in most cases and, at the same time, produces the same relative distribution of design forces to the substructure/foundation units as much more sophisticated models. It should be noted, however, that PPC deck beam bridges typically built on the State System have an overlay which would be considered to provide some degree continuity in the superstructure across the piers. As such, the transverse analysis and design methods described in this example are typically not applicable these bridges.

The connections between the superstructures and substructures are another aspect of simply supported PPC deck beam bridges which make them somewhat unique compared to other classes of bridges. Historically, the beams have been anchored to the abutments and piers with what would be considered a strong connection (two 1. in. ϕ dowel rods) that is not fuse like according to the Department's ERS strategy. Both rods will be engaged during each cycle of a seismic event while only one anchor bolt would engage in most other bridge types, and the rods are only loaded in pure shear compared to anchor bolts which are typically loaded in combined shear and tension (which produces an inherently weaker fuse like connection). It is probably imprudent, though, to require that anchorages for PPC deck beam bridges be designed to meet the same fuse level as their counterpart bridge types on the State and Local Systems. This is because there are other loads and considerations associated with PPC deck beam bridges which may override those for seismic loadings. I.e., fuses which have "amperage levels" that are too low may produce undesirable and/or unintended consequences not related to seismic loadings. As such, a value of 0.4 or C_{sm} (the design acceleration coefficient), whichever is smaller, times the dead load reaction should be used as the lateral (shear) load for design of connection rods compared to the value of 0.2 required for other classes of bridges typically built in Illinois. In addition, the smallest rod permitted shall be ¾ in. diameter.

The methods for analysis and design of multi-span simply supported PPC deck beam bridges in the longitudinal direction are straightforward and very similar to those for other classes of bridges built in Illinois.

It is common for simply supported PPC deck beam bridges to be located in rural areas of Illinois and/or areas where the ADT is low. When this is the case, bridges may be designed for the flexible option according to Section 3.15.8 of the Bridge Manual. The flexible option for design is presented for the bridge in this example. The recommended analysis and design techniques are also as simple as is reasonably possible for the heightened design accelerations associated with the 1000 yr. return period earthquake.

Discussions regarding how to meet the seismic requirements for single span PPC deck beam bridges are also interspersed at appropriate places throughout the example design presented below. Generally, single span structures should meet requirements for connections of superstructures to abutments (Part 12), and minimum seat widths (Part 13).

The example bridge is initially designed using HP piles. An overview of the bridge designed using metal shell piles is presented afterward.

1. Determination of Bridge Period – Transverse Direction

1.a. Weight of Bridge for Seismic Calculations

Refer to Example 1 Part 1.a. for discussion.

a.	Beams	27 in. x 48 in.			
		Est. weight per foot 1 beam	=	0.864 k/ft.	

		No. of beams	=	7
		Beam weight per foot	=	6.05 k/ft.
b.	Overlay	Non-cont. dead weight	=	0.050 ksf
		Width of deck	=	28 ft.
		Overlay weight per foot	=	1.4 k/ft.
c.	Rail	2 Rails	=	0.120 k/ft.
d.	Pier Cap	Length	=	29.5 ft.
		Width	=	3.5 ft.
		Height	=	2.5 ft.
		Pier cap weight		
		$29.5 \times 3.5 \times 2.5 \times 0.15\ ^{k}/_{ft.^3}$	=	38.7 kips
		Weight of 2 caps	=	77.4 kips
e.	Pier Piles	HP 12 x 53 weight per foot	=	0.053 k/ft.
		≈½ length above ground	=	4.5 ft.
		(assumes 1.5 ft. embedment)		
		Total no. of piles	=	14
		Total weight of piles	=	3.3 kips
		(piles may be ignored at designer's discretion)		
f.	Abut. Cap	Length	=	29.17 ft.
		Width	=	2.5 ft.
		Height	=	2.5 ft.
		Abut. cap weight		
		$29.17 \times 2.5 \times 2.5 \times 0.15\ ^{k}/_{ft.^3}$	=	27.3 kips
		Weight of 2 caps	=	54.6 kips
g.	Abut. Bckwall	Length	=	29.17 ft.
		Width	=	0.92 ft.
		Height	=	2.67 ft.
		Abut. backwall weight		
		$29.17 \times 0.92 \times 2.67 \times 0.15\ ^{k}/_{ft.^3}$	=	10.7 kips
		Weight of 2 backwalls	=	21.4 kips
h.	Total Weight	Length of Bridge	=	140 ft.
		a. + b. + c.	=	7.57 k/ft.
		or a. + b. + c.	=	1059.8 kips

d. + e. + f. + g.	=	156.7 kips
Total Weight	=	1216.5 kips
(Use for design)	≈	1220 kips

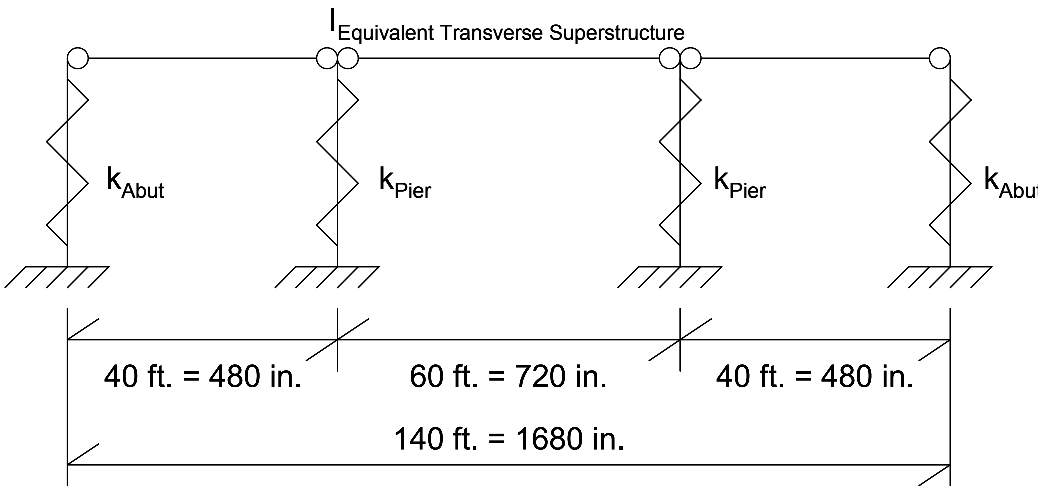
1.b. Global Transverse Structural Model of the Bridge

A straightforward finite element model of the bridge in this example can be constructed to determine the transverse period just as it is possible for most typical bridges built in Illinois. However, for most cases, a very simple approach to design and analysis is all that is absolutely necessary to calculate a reasonably accurate estimate of the transverse period (and the resulting subsequent distribution of the total base design shear to the substructure/foundation units). A short discussion of finite element modeling techniques which may be used for global analysis of multi-span simply supported PPC deck beam bridges is provided below for the benefit of designers who choose this analytical path. The discussion also serves to illustrate the logic behind the recommended simplified approach to transverse global analysis of simply supported multi-span PPC deck beam bridges.

It should be noted that PPC deck beam bridges built on the State System have an overlay which would be considered to provide some degree continuity in the superstructure across the piers. As such, the transverse analysis methods described in this example are typically not applicable these bridges.

A variation of the global analysis model used for the bridge in Example 1 which is suitable as a model for the bridge in this example is shown below. There are two primary differences between the analysis model of the Example 1 structure and the model employed for the current example bridge. The ends of each individual span are now pinned or are "released" in a finite element formulation, and the superstructure moment of inertia is some "equivalent" value. Engineering judgment is required to make a determination as to what is a reasonable moment of inertia to use for the superstructure given the loosely connected nature of the beams discussed above.

The piers and abutments in the model are represented by one spring finite element each. The stiffness calculations shown below are similar to those for the abutments in the Example 1 bridge. The superstructure is split up into 4 beam elements per span for a total of 12. A discussion on estimating the superstructure transverse moment of inertia (stiffness) is also provided below.



1.c. Transverse Pier Stiffness

The stiffness calculation method for the pile bent pier is directly analogous to Example 1 Part 1.d. except that the height of each pile (column in and above soil) is from the depth-of-fixity to the bottom of the pier cap. Note that the individual encasements of the piles are not considered structural. The soil type is Class D.

Piles	HP 12 x 53
Number of Piles	7
Weak Axis Pile Moment of Inertia (Typical Orientation for Illinois)	I_p = 127 in.^{4}
Steel Modulus of Elasticity	E_s = 29000 ksi
Pile Depth-of-Fixity (Fixed-Fixed) (from Geotechnical Analysis- see also Appendix C)	7.3 ft. = 87.6 in.
Pile Height Ground to Cap Bottom	6.0 ft. = 72 in.
Total Pile (Column) Height	159.6 in.
Stiffness of Pier	$k_{Pier} = \dfrac{(\text{no. piles}) \times 12 \times E_s \times I_p}{h_p^3}$

$$k_{Pier} = \frac{7 \times 12 \times 29000 \times 127}{159.6^3} = 76.1 \, ^{k}\!/_{in.}$$

1.d. Transverse Abutment Stiffness

The stiffness calculation method for the abutment is identical to Example 1 Part 1.d. The soil type is Class D.

Piles	HP 10 x 42
Number of Piles	5
Weak Axis Pile Moment of Inertia (Typical Orientation for Illinois)	I_p = 71.7 in.^{4}
Steel Modulus of Elasticity	E_s = 29000 ksi
Pile Depth-of-Fixity (Fixed-Fixed) (from Geotechnical Analysis- see also Appendix C)	6.7 ft. = 80.4 in.
Stiffness of Abutment	$k_{Abut} = \frac{(\text{no. piles}) \times 12 \times E_s \times I_p}{h_p^3}$

$$k_{Abut} = \frac{5 \times 12 \times 29000 \times 71.7}{80.4^3} = 240.0 \, ^{k}\!/_{in.}$$

1.e. Transverse Superstructure Stiffness

The beams should only be considered as contributing to the transverse moment of inertia of superstructure. However, the beams are only loosely tied together with several rods which are not post tensioned. A “fully effective” transverse moment of inertia for the superstructure can be calculated as given below.

Area of 1 Beam: 702 in.^{2}

Moment of Inertia of Deck:

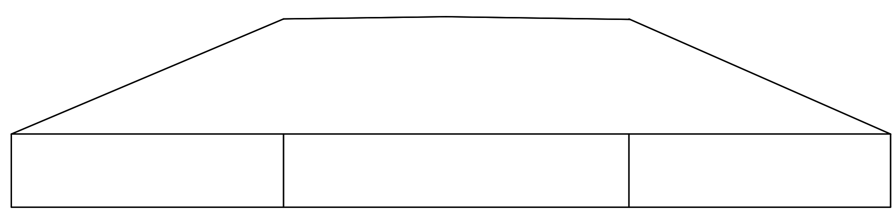
$$\sum_{i=1}^{n} A_{Beam} d_i^2, \; n = \text{no. of beams}, \; d = \text{distance from centroid (in.)}$$

$$= 702\left(2 \times 48^2 + 2 \times 96^2 + 2 \times 144^2\right) = 4.5 \times 10^7 \text{ in.}^4$$

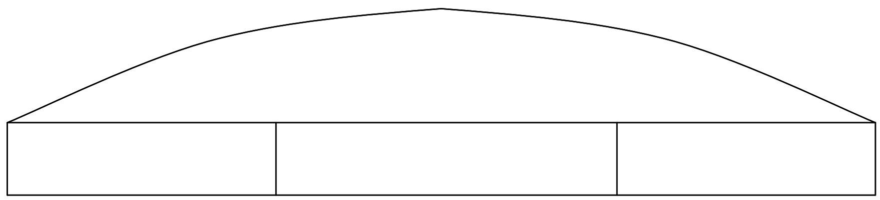
The equivalent moment of inertia may be estimated as about $\frac{1}{10}$th the fully effective for modeling purposes based upon engineering judgment or 4.5 x 10^6 $in.^4$ and is used for the finite element model in the current example. It should be noted that the calculated period is not overly sensitive to the value for superstructure moment of inertia actually employed in a global analysis model. However, the value used should be "in the ballpark" of reality. As a consequence, it is not recommended to use an overly stiff (or near rigid moment of inertia) or to simply assume the deck is a collection of beams which all act independently.

1.f. Finite Element and Simplified Transverse Period Determination

Using the stiffness values calculated above and a typical lumped mass formulation at the superstructure nodes, a period (T) of 0.72 sec. was obtained from the finite element model. The lumped mass assignments are compatible with the Uniform Load Method. The calculated shape of the first mode is as shown below.



The mode shape appears somewhat unusual due to the lack of continuity in the superstructure and the fact that the piers are flexible in relation to the abutments (by a factor of over three). The shape can be interpreted as a rough half sine wave which is comprised of 2 triangles and a rectangle. If the piers in the example bridge reached stiffnesses which are around ⅔ that of the abutments, the shape of the first mode of vibration would start to approach the familiar smoothly curved half sine wave shown below (similar to Example 1), and the period (T) would decrease from 0.72 sec. to 0.50 sec. It should be noted, however, that piers which are ⅔ as stiff as the abutments in the transverse direction would be considered out of the ordinary unless the piers were encased by a solid wall.



When either of the models above are subjected to an equal uniform load, both will have the same pier and abutment reactions due to the lack of continuity in the superstructure. Consequently, pier stiffnesses do not play a role in the distribution of total design base shear to substructure/foundation units of multi-span simply supported PPC deck beam bridges, but they do play a primary role in the actual magnitude of total base shear a structure is designed for. Pier stiffnesses are closely correlated to the transverse period of these bridges, and the period is used in conjunction with the design response spectrum to determine the fraction of a bridges weight (acceleration coefficient) to employ as the total base design shear.

Application of a 1 k/in. uniform load along the superstructure of the realistic finite element model (with flexible piers) results in a deflection at the center of the example bridge of 8.05 in., and at the piers it results in a deflection of 7.88 in. Using the Uniform Load Method (ULM), the equivalent stiffness of the bridge can be determined as,

$$k_{Bridge} = \frac{w \times L}{\delta_{Max}} = \frac{1 {}^{k}\!/_{in.} \times 140 \text{ ft.} \times 12 {}^{in.}\!/_{ft.}}{8.05 \text{ in.}} = 208.7 {}^{k}\!/_{in.}$$

And the period (T) can be estimated as,

Total Weight	1220 kips
Accel. of Gravity (g)	386.4 in./sec.^{2}
Bridge Stiffness	208.7 k/in.

$$T = 2\pi\sqrt{\frac{W}{g \times k_{Bridge}}} = 2\pi\sqrt{\frac{1220}{386.4 \times 208.7}} = 0.77 \text{ sec.}$$

As expected, the period calculated from the ULM is reasonably close to the more exacting calculation from the finite element model and is suitable for design purposes. Alternatively, the transverse period can also be calculated (estimated) using the Uniform Load Method assuming that the maximum deflection occurs at the piers as follows.

$$k_{Bridge} = \frac{w \times L}{\delta_{Pier}} = \frac{1 \, {}^{k}\!/\!_{in.} \times 140 \text{ ft.} \times 12 \, {}^{in}\!/\!_{ft.}}{7.88 \text{ in.}} = 213.2 \, {}^{k}\!/\!_{in.} \qquad \text{(Eq. 1)}$$

$$T = 2\pi \sqrt{\frac{W}{g \times k_{Bridge}}} = 2\pi \sqrt{\frac{1220}{386.4 \times 213.2}} = 0.76 \text{ sec.} \qquad \text{(Eq. 2)}$$

This alternative method also produces a reasonably accurate period which is suitable for design. Since the reactions for multi-span simply supported PPC deck beam bridges can be calculated according to the tributary span lengths framing into substructure/foundation units, the pier deflections for a 1 k/in. uniform load applied to the bridge can be calculated as follows.

Pier Reactions $$R_{P1} = R_{P2} = \left(\tfrac{1}{2} \times 40 \text{ ft.} + \tfrac{1}{2} \times 60 \text{ ft.}\right)\left(12 \, {}^{in}\!/\!_{ft.}\right)\left(1 \, {}^{k}\!/\!_{in.}\right) = 600 \text{ kips} \qquad \text{(Eq. 3)}$$

Pier Deflections $$\delta_{P1} = \delta_{P2} = \frac{600 \text{ kips}}{76.1 \, {}^{k}\!/\!_{in.}} = 7.88 \text{ in.} \qquad \text{(Eq. 4)}$$

As can be seen from the above derivation, sophisticated analytical techniques such as finite element modeling are not necessary to determine the period for most bridges which are in the same class as the bridge in the current example. A summary of the steps required to determine the period for most multi-span simply supported PPC deck beam bridges, typically up to 5 spans, is given below (simplified method) along with some discussion on their potential limitations.

1. Determine the transverse stiffnesses for each pier according to 1.c.

2. Determine the transverse stiffnesses for each abutment according to 1.d.

3. Compare the relative differences between the stiffnesses of the abutments and the piers. The abutments should be around two to three times stiffer than the piers at a minimum, but this is not absolutely required.

4. Compare the relative differences between the stiffnesses of the piers. If there are significant differences in stiffness from pier to pier, the period calculated from this method may be overly conservative, but it is still suitable for design purposes.

5. Using the stiffest pier, calculate the reaction according the tributary span length method described by Eq. 3 with a uniform load of 1 k/in.

6. Using the reaction calculated in Step 5, determine the deflection of the pier according the method described by Eq. 4.

7. Using the method described by Eq. 1, calculate the equivalent stiffness of the bridge.

8. Using the method describe by Eq. 2, calculate the period (T) of the bridge.

2. Determination of Bridge Period – Longitudinal Direction

2.a. Weight and Global Longitudinal Structural Model of the Bridge

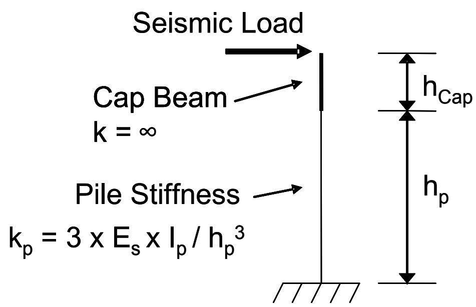
The methods required to determine the longitudinal period of multi-span simply supported PPC deck beam bridges are similar to those described in Example 1. The mass of the bridge used for calculation of the longitudinal period is the same as that used for the transverse direction. For bridges in the class of the current example structure, it is recommended that the longitudinal stiffnesses of all piers and abutments be used in the structural modeling and calculation of the period. Even though each span in the superstructure is simply supported; the ends of the spans are in close proximity, and seismic loads will be transmitted through the superstructure before the “fixed” dowel bars fuse and afterward through friction due to adequate seat widths. As such, the superstructure across the entire length of the bridge will essentially behave as one rigid body which transfers seismic loads to the substructures/foundations in a similar manner to bridges which have continuous superstructures. In addition, due to the nature of the configuration of bridges in this class (i.e. typically flexible piers), it is beneficial (and required in some cases) to rely upon the structural resistance of the abutments to absorb seismic loads in order to produce designs that are feasible.

2.b. Longitudinal Pier Stiffness

The piles (columns) for individually encased pile bent piers should be assumed to deflect as cantilevers which deform from “fixed ends” (at the depth-of-fixity in soil) to the bottom of the cap beam. The rigid body rotation of the cap should also be included in the stiffness determination. The equations and derivation below determine the stiffness, and the figure provides an illustration for guidance for one pile in a bent.

Pile Stiffness w/o Cap

$$k_p = \frac{3 \times E_s \times I_p}{h_p^3}\text{; where } h_p = \text{height taken as depth-of-fixity to bottom of cap}$$



Pile Stiffness w/ Cap

For a load P, the deflection at the top of a pile, δ_{Tp}, is:

$$k_p \times \delta_{Tp} = P \Rightarrow \delta_{Tp} = P/k_p = \frac{P \times h_p^3}{3 \times E_s \times I_p}$$

For a load P, the rotation at the top of a pile, θ_{Tp}, is:

$$\theta_{Tp} = \frac{P \times h_p^2}{2 \times E_s \times I_p}$$

The added deflection of the pier at the top of the cap beam, δ_A, is:

$$\delta_A = h_{Cap} \times \theta_{Tp}$$

The total deflection, δ_{TD}, is $\delta_{Tp} + \delta_A$ and the final pier pile long. stiffness then is:

$$k_{\text{Long Pier per Pile}} = P/\delta_{TD}$$

For a simple global stiffness analysis, the depth-of-fixity based upon deflection and not moment should be used for the fixed-pinned case. Appendix C provides depths-of-fixity in terms of moment. For the fixed-pinned case, the depths-of-fixity based upon deflection are about twice that for moment as discussed in Appendix C. The stiffness of the piers for the current example is given below.

Piles	HP 12 x 53
Number of Piles	7
Stong Axis Pile Moment of Inertia (Typical Orientation for Illinois)	I_p = 393 in.^{4}
Steel Modulus of Elasticity	E_s = 29000 ksi
Pile Depth-of-Fixity (Fixed-Pinned) (from Geotechnical Analysis- see also Appendix C)	2 x 4.9 ft. = 9.8 ft. = 117.6 in.
Pile Height Ground to Cap Bottom	6.0 ft. = 72 in.
Total Pile (Column) Height	189.6 in.
Stiffness of Piles w/o Cap	$k_P = \frac{(\text{no. piles}) \times 3 \times E_s \times I_p}{h_p^3}$

$$k_P = \frac{7 \times 3 \times 29000 \times 393}{189.6^3} = 35.1 \, ^{k}\!/_{\text{in.}}$$

Stiffness of Pier w/ Cap

δ_{TP} deflection at top of piles for a load P :

$$k_P \times \delta_{TP} = P \Rightarrow \delta_{TP} = P/35.1 = 0.028490P$$

θ_{TP}, rotation at top of piles for a load P :

$$\theta_{TP} = \frac{P \times 189.6^2}{7 \times 2 \times 29000 \times 393} = 0.00022530P$$

δ_A, the added deflection at the top of cap :

$$\delta_A = h_{Cap} \times \theta_{TP} = 30 \text{ in.} \times 0.00022530P = 0.0067590P$$

δ_{TD}, the total deflection :

$$\delta_{TD} = \delta_{TP} + \delta_A = 0.028490P + 0.0067590P = 0.035249P$$

The column stiffness with cap is :

$$k_{Long\,Pier} = P/\delta_{TD} = 28.4 \, ^{k}\!/_{\text{in.}}$$

2.c. Longitudinal Abutment Stiffness

The stiffness of the abutments is calculated in the same way as the piers with the exception that the piles are not exposed to air. As such, the abutments have shorter effective heights and are stiffer than the piers.

Piles	HP 10 x 42
Number of Piles	5
Stong Axis Pile Moment of Inertia (Typical Orientation for Illinois)	I_p = 210 in.^{4}
Steel Modulus of Elasticity	E_s = 29000 ksi
Pile Depth-of-Fixity (Fixed-Pinned) (from Geotechnical Analysis- see also Appendix C)	2 x 4.4 ft. = 8.8 ft. = 105.6 in.

Stiffness of Piles w/o Cap

$$k_A = \frac{(\text{no. piles}) \times 3 \times E_s \times I_p}{h_p^3}$$

$$k_A = \frac{5 \times 3 \times 29000 \times 210}{105.6^3} = 77.6 \ {}^{k}\!/_{in.}$$

Stiffness of Abutment w/ Cap

δ_{TA} deflection at top of piles for a load P :

$$k_A \times \delta_{TA} = P \Rightarrow \delta_{TA} = P/77.6 = 0.012887P$$

θ_{TA}, rotation at top of piles for a load P :

$$\theta_{TA} = \frac{P \times 105.6^2}{5 \times 2 \times 29000 \times 210} = 0.00018311P$$

δ_A, the added deflection at the top of cap :

$$\delta_A = h_{Cap} \times \theta_{TA} = 30 \text{ in.} \times 0.00018311P = 0.0054933P$$

δ_{TD}, the total deflection :

$$\delta_{TD} = \delta_{TA} + \delta_A = 0.012887P + 0.0054933P = 0.018380P$$

The column stiffness with cap is :

$$k_{Long\ Abut} = P/\delta_{TD} = 54.4 \ {}^{k}\!/_{in.}$$

2.d. Uniform Load Method Longitudinal Period Determination

The Uniform Load Method is used in the current example to straightforwardly calculate the longitudinal bridge period with the equation given below.

$$T = 2\pi\sqrt{\frac{M}{2\times k_{Long\,Pier} + 2\times k_{Long\,Abut}}};\text{where } M = \text{mass of bridge}$$

Mass of Bridge $M = \frac{\text{Weight of Bridge}}{g} = \frac{1220}{386.4} = 3.1573\ ^{k-sec.^2}/_{in.}$

Period $T = 2\pi\sqrt{\frac{3.1573}{2\times 28.4 + 2\times 54.4}} = 0.87\text{ sec.}$

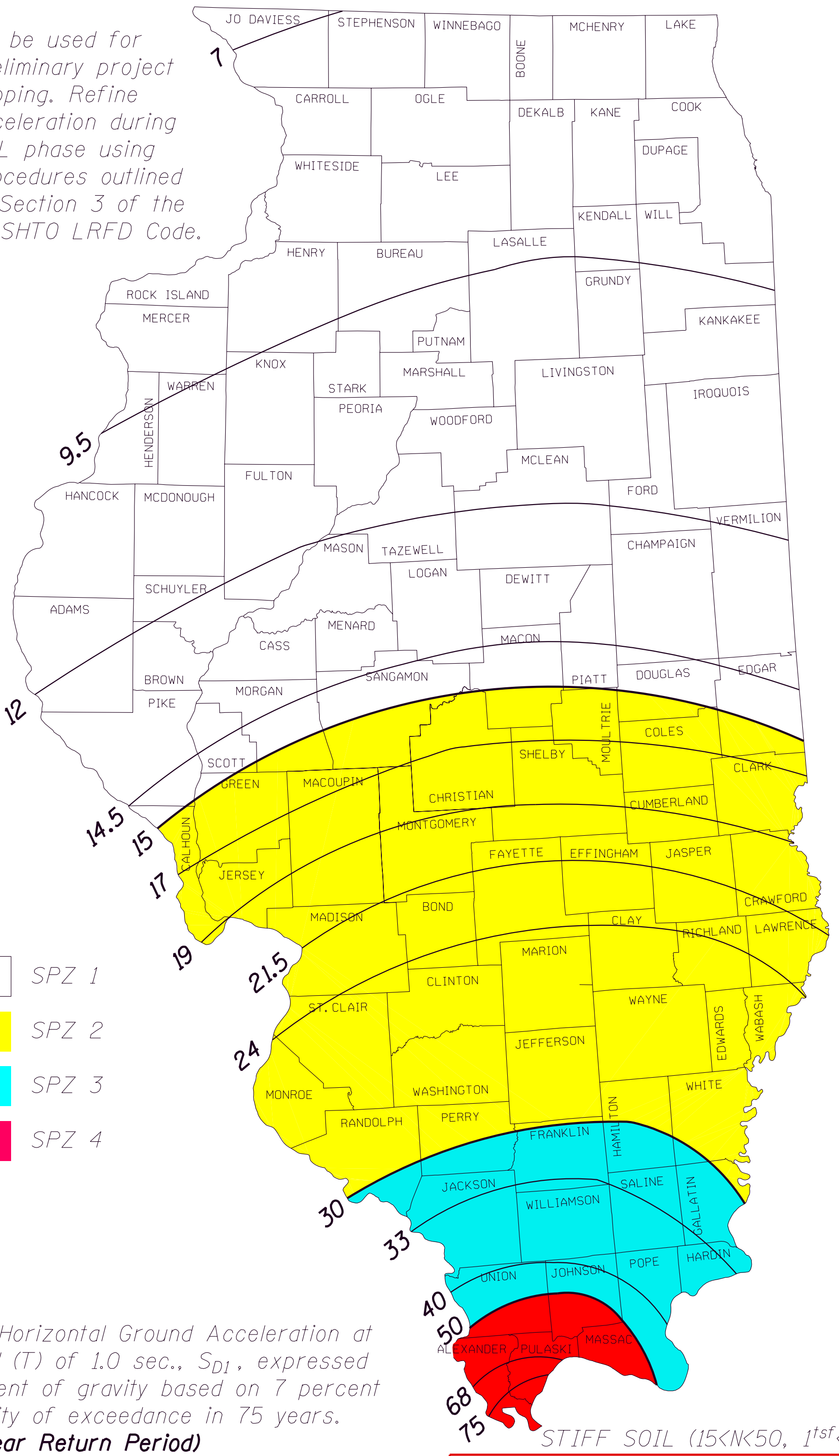
3. Determination of Base Shears – 1000 Year Design Earthquake Return Period

3.a. Design Response Spectrum (LRFD)

Reference Appendix 3.15.A of the Bridge Manual and Section 3 of the LRFD Code for more information on the formulation of the 1000 yr. Design Response Spectrum.

The bridge in this example is geographically located at the upper acceleration boundary of Zone 2 with Class D Soil (which is common in Southern Illinois). The map shown below is repeated from the Bridge Manual. It illustrates (approximately) the seismicity of Illinois for Class D soil. An examination of the map reveals that the bridge in the current example is being designed for moderate to high seismicity. This is also an area of the State for which PPC deck beam bridges are commonly constructed on the Local System.

Note: To be used for preliminary project scoping. Refine acceleration during TSL phase using procedures outlined in Section 3 of the AASHTO LRFD Code.



SPZ 1

SPZ 2

SPZ 3

SPZ 4

Map of Horizontal Ground Acceleration at a period (T) of 1.0 sec., S_{D1}, expressed as percent of gravity based on 7 percent probability of exceedance in 75 years.
(1000 Year Return Period)

Based on the U. S. Geological Survey 2007 maps for the 2008 AASHTO LRFD Code.

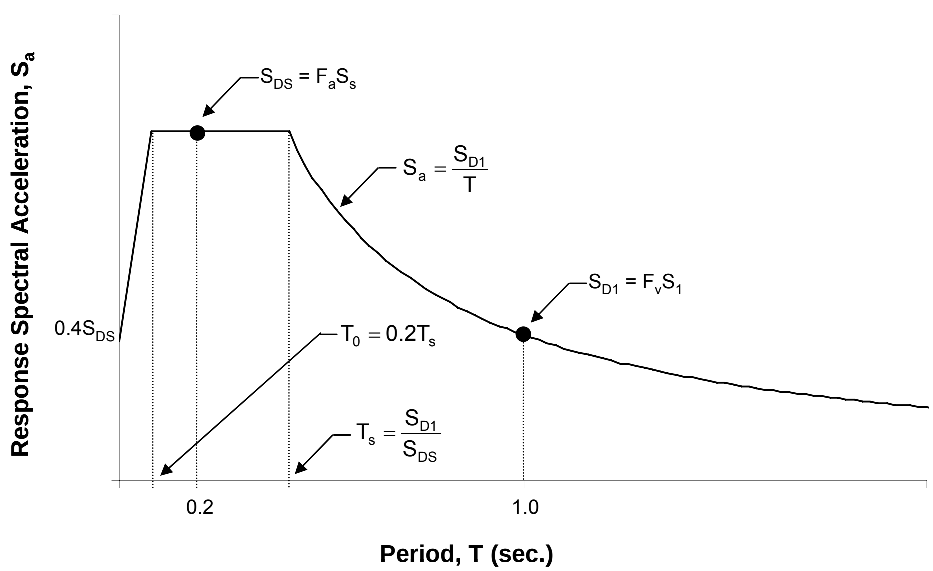
STIFF SOIL ($15<N<50$, $1^{tsf}<Q_u<2^{tsf}$)

SPECTRAL ACCELERATION
AT T=1.0 SEC. (S_{D1})
FOR SITE CLASS D SOIL (LRFD)

The following are the seismic design data for the current example bridge.

S_s (Short Period Acceleration)	0.50g (See LRFD Code Maps Article 3.10.2.1)
S_1 (1-Sec. Period Acceleration)	0.13g (See LRFD Code Maps Article 3.10.2.1)
Soil Type	Class D (In Upper 100 ft. of Soil Profile)
F_a (Short Period Soil Coef.)	1.40 (BM Table 3.15.A.2.3-1)
F_v (1-sec. Period Soil Coef.)	2.28 (BM Table 3.15.A.2.3-2)
S_{DS}	$F_a S_s = 1.4 \times 0.5 = 0.70g$
S_{D1}	$F_v S_1 = 2.28 \times 0.13 = 0.30g$
Seismic Performance Zone	2 $\left(0.15 < F_v S_1 \leq 0.30 \quad \text{BM Table 3.15.2 - 1}\right)$
Importance Category	Other (Flexible Option according to BM Section 3.15.8)

Definitions and a graphical representation of the design response spectrum (with approximate acceleration at zero sec. period) are given below.



T_s = $0.3/0.7 = 0.429$ sec.

T_0 = $0.2 \times 0.429 = 0.086$ sec.

Less than T_0, S_a = $0.6\frac{S_{DS}}{T_0}T + 0.4S_{DS} = 4.88T + 0.28$

Greater than T_s, S_a = $0.30/T$

(Where T = T_m, and S_a = C_{sm} in the LRFD Code)

Transverse Direction:

S_a (C_{sm} in the LRFD Code) = $0.30/0.76 = 0.39$ (T = 0.76 > T_s = 0.429 sec.)

Longitudinal Direction:

S_a (C_{sm} in the LRFD Code) = $0.30/0.87 = 0.34$ (T = 0.87 > T_s = 0.429 sec.)

Values beyond the "plateau" (analogous to 2.5A in the LFD Code) are not unexpected in the transverse and longitudinal directions for many typical multi-span simply supported PPC deck beam bridges.

<u>3.b. Transverse Base Shear</u>

Total Base Shear = $S_a \times \text{Wt. of Bridge} = 0.39 \times 1220 = 475.8\ \text{kips}$

Or $475.8\ \text{kips}/1680\ \text{in.} = 0.283\ \text{k/in.}$

As discussed above, the transverse seismic base shear is distributed to the piers and abutments according to the simple tributary span lengths which frame into these substructure/foundation units.

$V_{\text{Base Shear P (T)}}$ = $0.283\ \text{k/in.}\left(\tfrac{1}{2} \times 480\ \text{in.} + \tfrac{1}{2} \times 720\ \text{in}\right) = 169.8\ \text{kips}$

$V_{\text{Base Shear A (T)}}$ = $0.283\ \text{k/in.}\left(\tfrac{1}{2} \times 480\ \text{in.}\right) = 67.9\ \text{kips}$

<u>3.c. Longitudinal Base Shear</u>

Total Base Shear = $S_a \times \text{Wt. of Bridge} = 0.34 \times 1220 = 414.8\ \text{kips}$

For this example, the longitudinal seismic base shear is distributed to the piers and abutments according to the relative stiffnesses of these units.

$V_{Base\ Shear\ P\ (L)}$ = $\frac{28.4\ ^{k}/_{in.}}{(2 \times 28.4 + 2 \times 54.4)\ ^{k}/_{in.}} \times 414.8\ \text{kips} = 71.1\ \text{kips}$

$V_{Base\ Shear\ A\ (L)}$ = $\frac{54.4\ ^{k}/_{in.}}{(2 \times 28.4 + 2 \times 54.4)\ ^{k}/_{in.}} \times 414.8\ \text{kips} = 136.3\ \text{kips}$

4. Frame Analysis and Pile (Columnar) Seismic Forces for Pile Bents

4.a. Pier Forces – Dead Load

Dead Load of Superstructure

(Use Wt. from Previous Calculations) = 1220 kips

Bridge Length = 140 ft.

Dead Load Per ft. of Bridge = $^{1220}/_{140} = 8.714\ ^{k}/_{ft.}$

Dead Load Per Pier (Use Statics) = $8.714 \times \left(\frac{1}{2} L_{OuterSpan} + \frac{1}{2} L_{CenterSpan}\right)$

= $8.714 \times \left(\frac{1}{2} \times 40 + \frac{1}{2} \times 60\right) = 435.7\ \text{kips}$

No. of Piles (Columns) Per Pier = 7

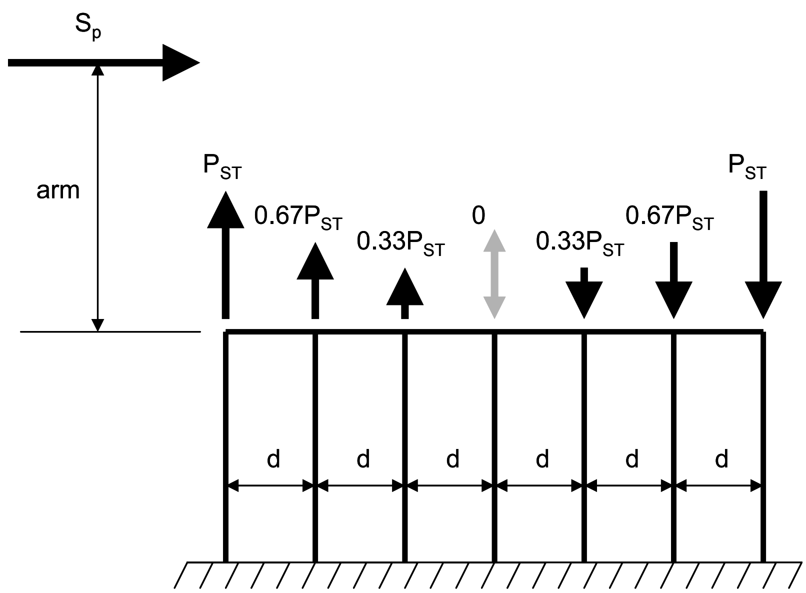
Dead Load Per Pile = $^{435.7}/_{7} = 62.2\ \text{kips}$

Added Dead Load for Bot. Half of 1 Col.

Not Considered in Pt. 1.a. Sub-Pt. e. = Neglect as not significant

4.b. Pier Forces – Transverse Overturning

The seismic base shear at each pier theoretically acts through the centroid of the superstructure. It is acceptable to assume/approximate that the centroid acts at the top of the beams. From statics, shown below, an "overturning moment" produces axial compression and tension across the bent assuming a connection which resists uplift between the superstructure and the substructures. The seismic base shear also produces "frame action" forces in the piles (columns) of the bent and acts at the bottom of the cap (the "eccentricity" or "arm" of the base shear is taken into account through consideration of the overturning moment) Frame action force analysis is given in the following section. Appendix A contains overturning moment solutions for bents with 2 to 13 columns. In reality, for multi-span simply supported PPC deck beam structures, the axial compressive forces predicted from the simple overturning moment solutions are probably more accurate than the predicted tensile forces due to the nature of the connection between the superstructures and substructures for this class of bridges. However, the solutions give results which are suitable for design purposes.



$$M = S_p \times arm$$

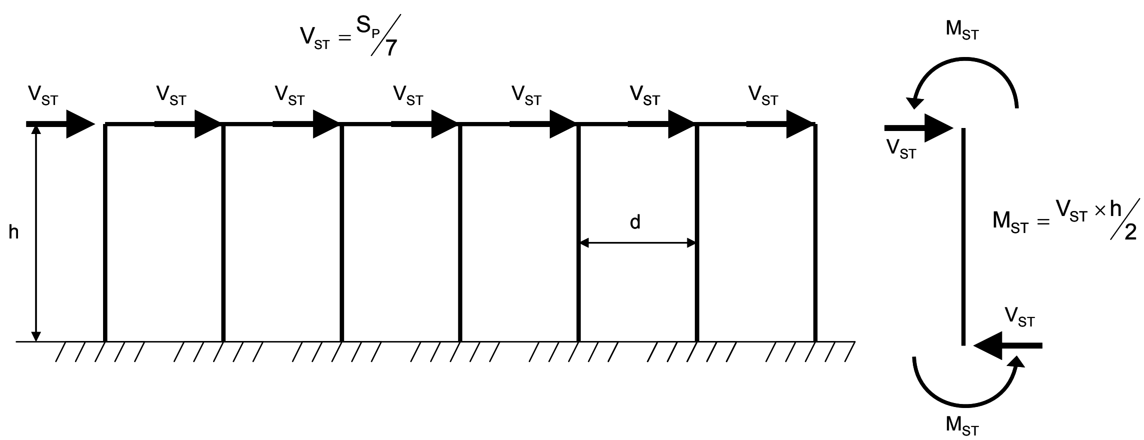
$$M = 2(P_{ST} \times 3d) + 2(0.67P_{ST} \times 2d) + 2(0.33P_{ST} \times d)$$

$$\therefore P_{ST} = 0.107 \, M/d$$

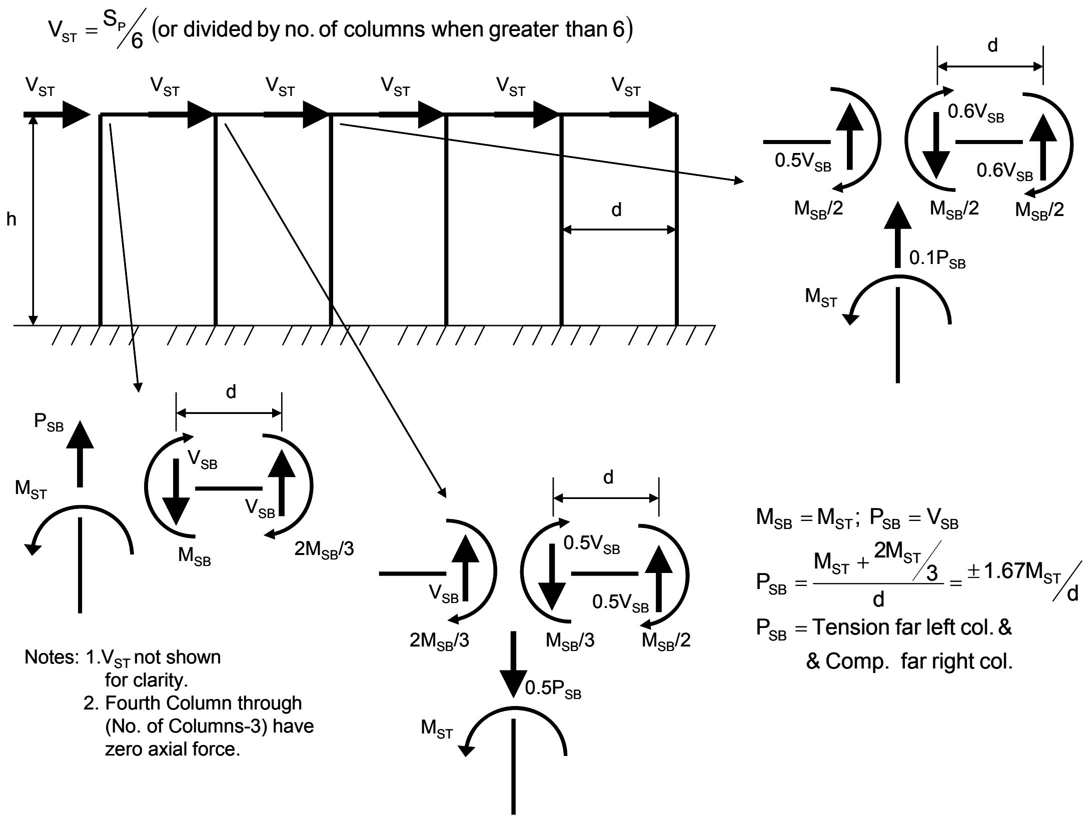
S_p (Base Shear at Pier)	=	169.8 kips
arm (Base Shear Eccentricity)		
Cap Height + Beam Height	=	$2.5 + 27/12 = 4.75$ ft.
d (Center-to-Center Pile Distance)	=	4 ft. – 2 in. = 4.17 ft.
M (Overturning Moment)	=	$169.8 \times 4.75 = 806.6$ k - ft.
P_{ST} (Maximum Axial Columnar Force)	=	$0.107 \times 806.6/4.17 = 20.7$ kips

4.c. Pier Forces – Transverse Frame Action

Taking account of the overturning moment "transfers" the seismic base shear at the pier to the tops of the piles (columns). This shear produces moments, shears and axial forces in each pile of the bent through "frame action." Free body diagram solutions for these seismic forces are shown below. The determination of moment and shear in each column is more straightforward than for axial force. The simple solutions for moment in the piles are very accurate while conservative solutions for axial force due frame action are emphasized for the critical outside piles. Appendix B contains frame action columnar axial force solutions for bents with 2 to 6 or more columns.



S_p (Base Shear at Pier)	=	169.8 kips
Column Height (Fixity to Cap Bot.)	=	13.3 ft.
V_{ST} (Shear Per Pile)	=	$169.8/7 = 24.3$ kips
M_{ST} (Moment Per Pile)	=	$24.3 \times 13.3/2 = 161.6$ k-ft.



M_{ST} (Moment Per Pile)	=	161.6 k-ft.
d (Center-to-Center Pile Distance)	=	4.17 ft.
P_{SB} (Maximum Axial Pile Force)	=	$1.67 \times 161.6 / 4.17 = 64.7$ kips

<u>4.d. Pier Forces – Longitudinal Cantilever</u>

Only simple cantilever statics is required to determine the seismic shear and moment in the longitudinal direction.

S_L (Base Shear at Pier)	=	71.1 kips
Column Height (Fixity Based on Moment to Cap Bot.)	=	4.9 ft. + 6.0 ft. = 10.9 ft.
Cap Beam Height	=	2.5 ft.
V_{SL} (Shear Per Pile)	=	$71.1/7 = 10.2$ kips
$M_{ColBot(SLB)}$ (Moment Per Pile)	=	$10.2 \times (10.9 + 2.5) = 136.7$ k-ft.

5. Frame Analysis and Pile (Columnar) Seismic Forces for Abutments

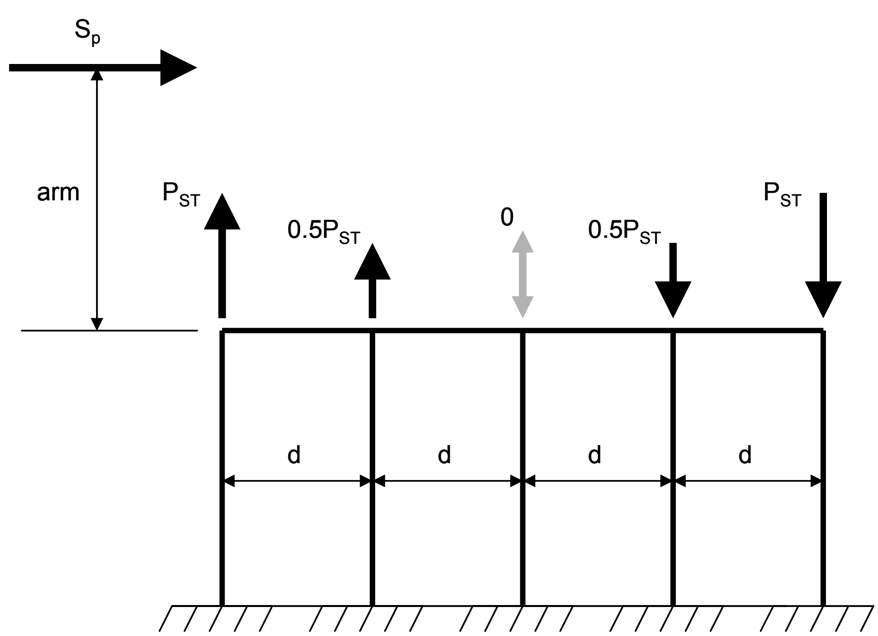
The analysis of the abutments for seismic forces is directly analogous to that of the piers.

5.a. Abutment Forces – Dead Load

Dead Load of Superstructure

(Use Wt. from Previous Calculations)	=	1220 kips
Bridge Length	=	140 ft.
Dead Load Per ft. of Bridge	=	$1220/140 = 8.714\ ^{k}/_{ft.}$
Dead Load Per Abut. (Use Statics)	=	$8.714 \times \left(\frac{1}{2} L_{OuterSpan}\right)$
	=	$8.714 \times \left(\frac{1}{2} \times 40\right) = 174.3\ \text{kips}$
No. of Piles (Columns) Per Abut.	=	5
Dead Load Per Pile	=	$174.3/5 = 34.9\ \text{kips}$

5.b. Abutment Forces – Transverse Overturning



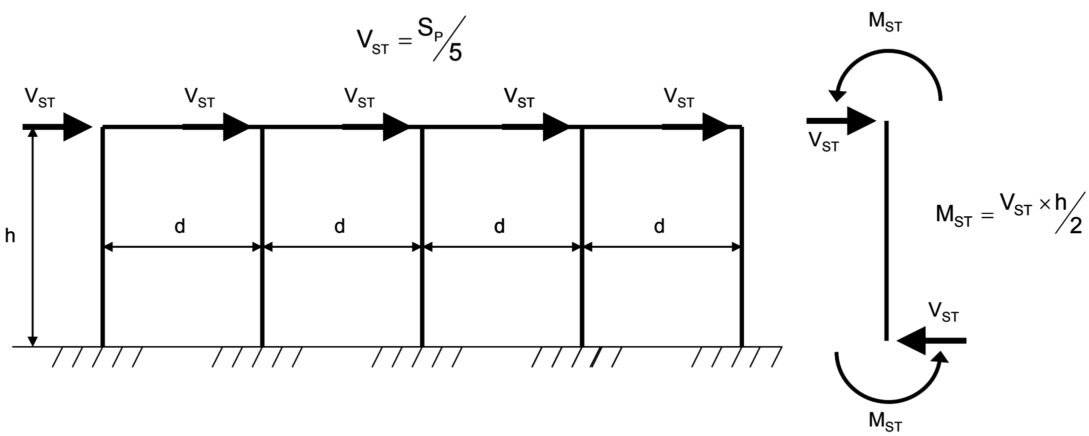
$$M = S_p \times arm$$

$$M = 2(P_{ST} \times 2d) + 2(\tfrac{1}{2} P_{ST} \times d)$$

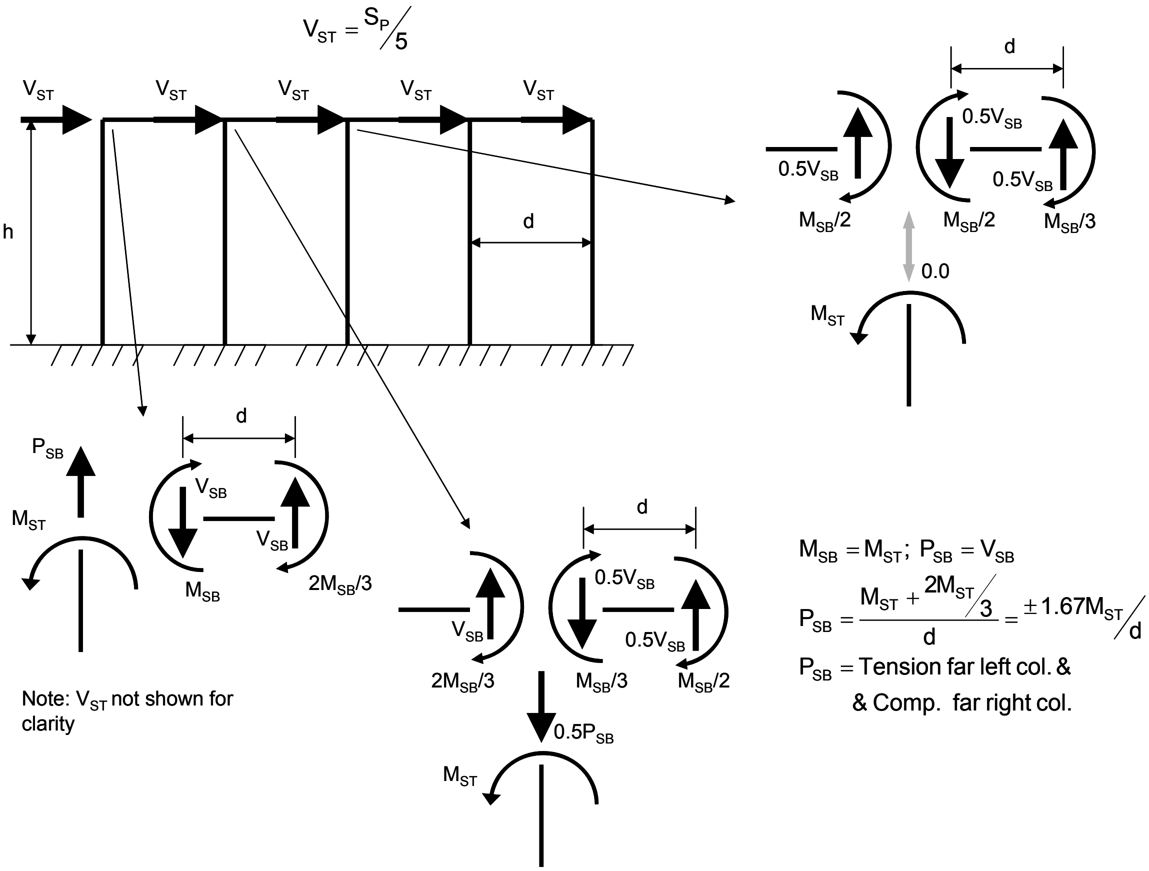
$$\therefore P_{ST} = \frac{1}{5} M/d$$

S_p (Base Shear at Abutment)	=	67.9 kips
arm (Base Shear Eccentricity)		
Cap Height + Beam Height	=	$2.5 + 27/12 = 4.75$ ft.
d (Center-to-Center Pile Distance)	=	6 ft. – 3 in. = 6.25 ft.
M (Overturning Moment)	=	$67.9 \times 4.75 = 322.5$ k - ft.
P_{ST} (Maximum Axial Columnar Force)	=	$\frac{1}{5} \times 322.5/6.25 = 10.3$ kips

5.c. Abutment Forces – Transverse Frame Action



S_p (Base Shear at Abutment)	=	67.9 kips
Column Height (Fixity to Cap Bot.)	=	6.7 ft.
V_{ST} (Shear Per Pile)	=	$67.9/5 = 13.6$ kips
M_{ST} (Moment Per Pile)	=	$13.6 \times 6.7/2 = 45.6$ k - ft.



M_{ST} (Moment Per Pile)	=	45.6 k-ft.
d (Center-to-Center Pile Distance)	=	6.25 ft.
P_{SB} (Maximum Axial Pile Force)	=	$1.67 \times 45.6 / 6.25 = 12.2$ kips

5.d. Abutment Forces – Longitudinal Cantilever

S_L (Base Shear at Abutment)	=	136.3 kips
Column Height (Fixity Based on Moment to Cap Bot.)	=	4.4 ft.
Cap Beam Height	=	2.5 ft.
V_{SL} (Shear Per Pile)	=	$136.3/5 = 27.3$ kips
$M_{ColBot(SLB)}$ (Moment Per Pile)	=	$27.3 \times (4.4 + 2.5) = 188.4$ k-ft.

6. Seismic Design Forces for Pile Bent Including R-Factor, P-Δ, and Combination of Orthogonal Forces

6.a. R-Factor

R-Factors should only be used to reduce the moments calculated from the base shears of an "elastic" analysis as was conducted above. As recommended in the Bridge Manual (Section 3.15.8) an R-Factor of 3.5 will be used for the pile bents in this example. This coincides with classifying the bridge as "Other" according to the LRFD Code.

6.b. P-Δ

See Example 1 Part 6.b. for discussion on this topic. For the current example bridge, a 5% amplification of bending moments will be used to account for P-Δ effects.

6.c. Summary and Combination of Orthogonal Column Forces Used for Design

The forces on the two exterior piles (columns) in the example bridge are focused on for design because they experience the most extreme earthquake forces. The pier piles should be designed for the possibility of earthquake accelerations which can be in opposite transverse directions and opposite longitudinal directions. They are also required to be designed for the cases "mostly longitudinal and some transverse accelerations" (Longitudinal Dominant – Load Case 1) and "some longitudinal and mostly transverse accelerations" (Transverse Dominant – Load Case 2).

Since the bridge in this example is not skewed, the combination of orthogonal forces used for design is straightforward. More complex cases with skew and non-round columns are considered in Examples 2 and 3 of this design guide. The equations below present the basic method for combination of orthogonal forces.

Load Case 1 (Longitudinal Dominant)

$$V_z^D = 1.0\left|V_z^L\right| + 0.3\left|V_z^T\right|$$

Load Case 2 (Transverse Dominant)

$$V_z^D = 0.3\left|V_z^L\right| + 1.0\left|V_z^T\right|$$

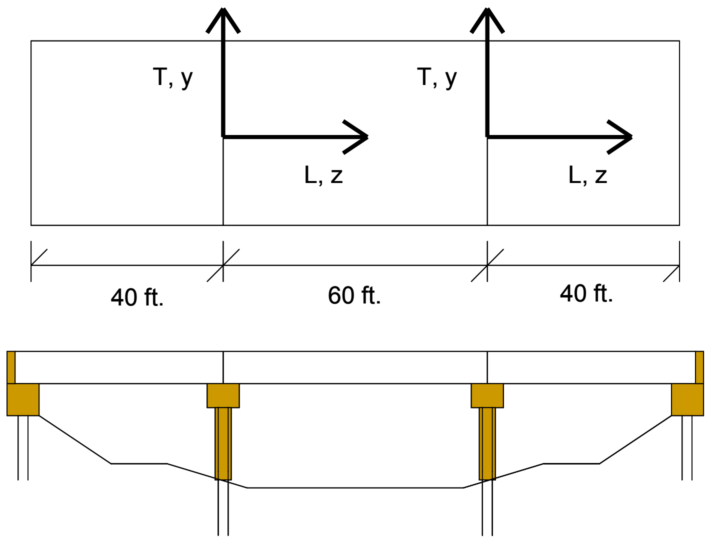
$$V_y^D = 1.0|V_y^L| + 0.3|V_y^T| \qquad V_y^D = 0.3|V_y^L| + 1.0|V_y^T|$$

$$M_z^D = 1.0|M_z^L| + 0.3|M_z^T| \qquad M_z^D = 0.3|M_z^L| + 1.0|M_z^T|$$

$$M_y^D = 1.0|M_y^L| + 0.3|M_y^T| \qquad M_y^D = 0.3|M_y^L| + 1.0|M_y^T|$$

$$P^D = 1.0|P^L| + 0.3|P^T| \qquad P^D = 0.3|P^L| + 1.0|P^T|$$

For the bridge in this example the Longitudinal- and z-axes, and Transverse- and y-axes coincide as shown below.



Shown below are the Load Case 1 and Load Case 2 forces used for seismic design with R-Factor (3.5), P-Δ amplification (1.05), and axial dead load (62.2 kips) effects all considered.

Load Case 1 – Longitudinal Dominant (per pile):

$$V_z^D = 1.0|V_z^L| + 0.3|V_z^T| = 1.0|V_{SL}| + 0.3|0| = 1.0|10.2| = 10.2 \text{ kips}$$

$$V_y^D = 1.0|V_y^L| + 0.3|V_y^T| = 1.0|0| + 0.3|V_{ST}| = 0.3|24.3| = 7.3 \text{ kips}$$

$$M_z^D = 1.0|M_z^L| + 0.3|M_z^T| = 1.0|0| + 0.3\left|\frac{1.05 \times M_{ST}}{3.5}\right| = 0.3\left|\frac{1.05 \times 161.6}{3.5}\right| = 14.5 \text{ k-ft.}$$

$$M_y^D = 1.0|M_y^L| + 0.3|M_y^T| = 1.0\left|\frac{1.05 \times M_{SLB}}{3.5}\right| + 0.3|0| = 1.0\left|\frac{1.05 \times 136.7}{3.5}\right| = 41.0 \text{ k-ft.}$$

$$P^D = 1.0|P^L| + 0.3|P^T| \rightarrow P^D = P_{Dead} + 1.0|0| \pm 0.3|P_{ST} + P_{SB}| = 62.2 \pm 0.3|20.7 + 64.7| =$$
$$= 36.6 \text{ and } 87.8 \text{ kips}$$

Note that the Department recommends a load factor of 1.0 be used for dead loads for LRFD.

Load Case 2 – Transverse Dominant (per pile):

$$V_z^D = 0.3|V_z^L| + 1.0|V_z^T| = 0.3|V_{SL}| + 1.0|0| = 0.3|10.2| = 3.1 \text{ kips}$$

$$V_y^D = 0.3|V_y^L| + 1.0|V_y^T| = 0.3|0| + 1.0|V_{ST}| = 1.0|24.3| = 24.3 \text{ kips}$$

$$M_z^D = 0.3|M_z^L| + 1.0|M_z^T| = 0.3|0| + 1.0\left|1.05 \times M_{ST} / 3.5\right| = 1.0\left|1.05 \times 161.6 / 3.5\right| = 48.5 \text{ k-ft.}$$

$$M_y^D = 0.3|M_y^L| + 1.0|M_y^T| = 0.3\left|1.05 \times M_{SLB} / 3.5\right| + 1.0|0| = 0.3\left|1.05 \times 136.7 / 3.5\right| = 12.3 \text{ k-ft.}$$

$$P^D = 0.3|P^L| + 1.0|P^T| \rightarrow P^D = P_{Dead} + 0.3|0| \pm 1.0|P_{ST} + P_{SB}| = 62.2 \pm 1.0|20.7 + 64.7| =$$
$$= -23.2 \text{ (Ten.) and } 147.6 \text{ kips}$$

7. *Seismic Design Forces for Abutment Including R-Factor, P-Δ, and Combination of Orthogonal Forces*

7.a. R-Factor

As recommended in the Bridge Manual (Section 3.15.8) an R-Factor of 1.5 will be used for the piles supporting the stub abutments in this example. This coincides with classifying the bridge as “Other” according to the LRFD Code.

7.b. P-Δ

P-Δ effects should be ignored for piles at abutments.

7.c. Summary and Combination of Orthogonal Column Forces Used for Design

The method for combining of orthogonal forces for the abutments is similar to that for the piers. Shown below are the Load Case 1 and Load Case 2 forces used for seismic design with R-Factor (1.5), P-Δ amplification (1.00), and axial dead load (34.9 kips) effects all considered.

Load Case 1 – Longitudinal Dominant (per pile):

$$V_z^D = 1.0\left|V_z^L\right| + 0.3\left|V_z^T\right| = 1.0\left|V_{SL}\right| + 0.3\left|0\right| = 1.0\left|27.3\right| = 27.3 \text{ kips}$$

$$V_y^D = 1.0\left|V_y^L\right| + 0.3\left|V_y^T\right| = 1.0\left|0\right| + 0.3\left|V_{ST}\right| = 0.3\left|13.6\right| = 4.1 \text{ kips}$$

$$M_z^D = 1.0\left|M_z^L\right| + 0.3\left|M_z^T\right| = 1.0\left|0\right| + 0.3\left|\frac{1.00 \times M_{ST}}{2.0}\right| = 0.3\left|\frac{1.00 \times 45.6}{1.5}\right| = 9.1 \text{ k-ft.}$$

$$M_y^D = 1.0\left|M_y^L\right| + 0.3\left|M_y^T\right| = 1.0\left|\frac{1.00 \times M_{SLB}}{2.0}\right| + 0.3\left|0\right| = 1.0\left|\frac{1.00 \times 188.4}{1.5}\right| = 125.6 \text{ k-ft.}$$

$$P^D = 1.0\left|P^L\right| + 0.3\left|P^T\right| \rightarrow P^D = P_{Dead} + 1.0\left|0\right| \pm 0.3\left|P_{ST} + P_{SB}\right| = 34.9 \pm 0.3\left|10.3 + 12.2\right| =$$
$$= 28.2 \text{ and } 41.7 \text{ kips}$$

Note that the Department recommends a load factor of 1.0 be used for dead loads for LRFD.

Load Case 2 – Transverse Dominant (per pile):

$$V_z^D = 0.3\left|V_z^L\right| + 1.0\left|V_z^T\right| = 0.3\left|V_{SL}\right| + 1.0\left|0\right| = 0.3\left|27.3\right| = 8.2 \text{ kips}$$

$$V_y^D = 0.3\left|V_y^L\right| + 1.0\left|V_y^T\right| = 0.3\left|0\right| + 1.0\left|V_{ST}\right| = 1.0\left|13.6\right| = 13.6 \text{ kips}$$

$$M_z^D = 0.3\left|M_z^L\right| + 1.0\left|M_z^T\right| = 0.3\left|0\right| + 1.0\left|\frac{1.00 \times M_{ST}}{2.0}\right| = 1.0\left|\frac{1.00 \times 45.6}{1.5}\right| = 30.4 \text{ k-ft.}$$

$$M_y^D = 0.3\left|M_y^L\right| + 1.0\left|M_y^T\right| = 0.3\left|\frac{1.00 \times M_{SLB}}{2.0}\right| + 1.0\left|0\right| = 0.3\left|\frac{1.00 \times 188.4}{1.5}\right| = 37.7 \text{ k-ft.}$$

$$P^D = 0.3\left|P^L\right| + 1.0\left|P^T\right| \rightarrow P^D = P_{Dead} + 0.3\left|0\right| \pm 1.0\left|P_{ST} + P_{SB}\right| = 34.9 \pm 1.0\left|10.3 + 12.2\right| =$$
$$= 12.4 \text{ and } 57.4 \text{ kips}$$

8. Combined Axial Force and Bi-Axial Bending Structural Capacity Check for Piles in Bents

The piles should be checked for structural capacity using Articles 6.8 (Tension Members) or Article 6.9 (Compression Members) of the LRFD Code for combined axial force and bi-axial bending. Note that all ϕ factors according to LRFD Article 6.5.5 shall be taken as 1.0 for extreme events.

Geotechnical considerations for the design of piles to resist seismic loadings are provided in Sections 3.10 and 3.15 of the Bridge Manual as well as Design Guide 3.10.1. Guidance on liquefaction, lateral pile resistance in soil, pullout from the soil, etc. are among the considerations addressed.

8.a. Load Case 1 – Longitudinal Dominant

Both axial forces for Load Case 1 are compressive. LRFD Article 6.9.2.2 contains the provisions for checking axial compression in combination with bi-axial bending and shall satisfy,

For $\frac{P_u}{P_r} < 0.2$,

$$\frac{P_u}{2.0P_r} + \left(\frac{M_{ux}}{M_{rx}} + \frac{M_{uy}}{M_{ry}}\right) \leq 1.0 \qquad \text{(Eq. 6.9.2.2-1)}$$

Otherwise,

$$\frac{P_u}{P_r} + \frac{8}{9}\left(\frac{M_{ux}}{M_{rx}} + \frac{M_{uy}}{M_{ry}}\right) \leq 1.0 \qquad \text{(Eq. 6.9.2.2-2)}$$

Where:

M_{ux} = design seismic moment about y-axis of bent (k-ft. or k-in.)

M_{uy} = design seismic moment about z-axis of bent (k-ft. or k-in.)

M_{rx} = F_yS_x, yield strength times section modulus about strong axis of pile (k-ft. or k-in.)

M_{ry} = F_yS_y, yield strength times section modulus about weak axis of pile (k-ft. or k-in.)

P_u = design seismic compressive force (kips)

P_r = allowable compressive resistance according to LRFD Article 6.9.2.1 (kips)

$P_r = \phi P_n$ (Eq. 6.9.2.1-1)

Where:

For $\lambda \leq 2.25$,

$P_n = 0.66^{\lambda} F_y A_s$ (Eq. 6.9.4.1-1)

Otherwise,

$$P_n = \frac{0.88 F_y A_s}{\lambda}$$ (Eq. 6.9.4.1-2)

In which:

$$\lambda = \left(\frac{K\ell}{r_s \pi}\right)^2 \frac{F_y}{E}$$ (Eq. 6.9.4.1-3)

Where:

ϕ	=	1.0
A_s	=	gross area of section (in.^{2})
F_y	=	yield strength of steel (ksi)
E	=	modulus of elasticity of steel (ksi)
K	=	effective length factor
ℓ	=	unbraced length (in.)
r_s	=	radius of gyration about plane of buckling (in.)

For Load Case 1 (HP 12 x 53),

$M_{ux} = M_y^D$	=	41.0 k-ft.
$M_{uy} = M_z^D$	=	14.5 k-ft.
$M_{rx} = F_y S_x = 50\,\text{ksi} \times 66.8\,\text{in.}^3 \times {}^{1\,\text{ft}}/_{12\,\text{in.}}$	=	278.3 k-ft.
$M_{ry} = F_y S_y = 50\,\text{ksi} \times 21.1\,\text{in.}^3 \times {}^{1\,\text{ft}}/_{12\,\text{in.}}$	=	87.9 k-ft.
$P_u = P^D$ (max.)	=	87.8 kips

$P_r = \phi P_n = 1.0 P_n$

$$\lambda = \left(\frac{K\ell}{r_s \pi}\right)^2 \frac{F_y}{E}$$

K	=	1.5 (≈ avg. about weak and strong axis pile)
ℓ	=	159.6 in. (about weak axis, would be 130.8 in. about strong axis taking depth-of-fixity based upon moment)
F_y	=	50 ksi
E	=	29000 ksi
r_s	=	2.86 in. (about weak axis)
A_s	=	15.5 in.^{2}

$$\lambda = \left(\frac{1.5 \times 159.6}{2.86 \times \pi}\right)^2 \frac{50}{29000} = \quad 1.22$$

$\lambda \le 2.25$,

$\therefore P_r = (1.0)0.66^{\lambda}F_yA_s = (1.0)\,0.66^{1.22} \times 50 \times 15.5 =$ 466.8 kips

And,

$$\frac{P_u}{P_r} = \frac{87.8}{466.8} = 0.19 < 0.2\,,$$

$$\therefore \ \frac{P_u}{2.0P_r} + \left(\frac{M_{ux}}{M_{rx}} + \frac{M_{uy}}{M_{ry}}\right) = \frac{87.8}{2.0 \times 466.8} + \left(\frac{41.0}{278.3} + \frac{14.5}{87.9}\right) = 0.41 \le 1.0 \ \text{ OK}$$

<u>8.b. Load Case 2 – Transverse Dominant</u>

One design axial force for Load Case 2 is compressive and one is tensile. However, the tensile load is much smaller that the compressive force in absolute magnitude, and the allowable tensile force in pure tension is generally greater than for compression. The provisions for axial tension combined with bi-axial bending are similar to those for axial compression. Therefore, only LRFD Article 6.9.2.2 need be checked for Load Case 2.

For Load Case 2 (HP 12 x 53),

$M_{ux} = M^D_y$	=	12.3 k-ft.
$M_{uy} = M^D_z$	=	48.5 k-ft.
$M_{rx} = F_yS_x = 50\,\text{ksi} \times 66.8\,\text{in.}^3 \times {}^{1\text{ft}}\!/_{12\,\text{in.}}$	=	278.3 k-ft.

$M_{ry} = F_yS_y = 50\,\text{ksi} \times 21.1\,\text{in.}^3 \times {}^{1\text{ft}}\!/_{12\,\text{in.}}$ = 87.9 k-ft.

$P_u = P^D$ (max.) = 147.6 kips

$P_r = \phi P_n = 1.0P_n$

$$\lambda = \left(\frac{K\ell}{r_s\pi}\right)^2 \frac{F_y}{E}$$

K = 1.5 (≈ avg. about weak and strong axis pile)

ℓ = 159.6 in. (about weak axis, would be 130.8 in. about strong axis taking depth-of-fixity based upon moment)

F_y = 50 ksi

E = 29000 ksi

r_s = 2.86 in. (about weak axis)

A_s = 15.5 in.^{2}

$$\lambda = \left(\frac{1.5 \times 159.6}{2.86 \times \pi}\right)^2 \frac{50}{29000} = 1.22$$

$\lambda \le 2.25$,

$\therefore P_r = (1.0)0.66^{\lambda}F_yA_s = (1.0)\,0.66^{1.22} \times 50 \times 15.5 =$ 466.8 kips

And,

$$\frac{P_u}{P_r} = \frac{147.6}{466.8} = 0.32 > 0.2\,,$$

$$\therefore \frac{P_u}{P_r} + \frac{8}{9}\left(\frac{M_{ux}}{M_{rx}} + \frac{M_{uy}}{M_{ry}}\right) = \frac{147.6}{466.8} + \frac{8}{9}\left(\frac{12.3}{278.3} + \frac{48.5}{87.9}\right) = 0.85 \le 1.0 \text{ OK}$$

9. Combined Axial Force and Bi-Axial Bending Structural Capacity Check for Piles in Abutments

The piles in the abutments should be checked with the same LRFD provisions as the piles in the piers.

9.a. Load Case 1 – Longitudinal Dominant

For Load Case 1 (HP 10 x 42),

$M_{ux} = M_y^D$ = 125.6 k-ft.

$M_{uy} = M_z^D$ = 9.1 k-ft.

$M_{rx} = F_yS_x = 50\,\text{ksi} \times 43.4\,\text{in.}^3 \times {}^{1\text{ft}}\!/_{12\,\text{in.}}$ = 180.8 k-ft.

$M_{ry} = F_yS_y = 50\,\text{ksi} \times 14.2\,\text{in.}^3 \times {}^{1\text{ft}}\!/_{12\,\text{in.}}$ = 59.2 k-ft.

$P_u = P^D$ (max.) = 41.7 kips

$P_r = \phi P_n = 1.0P_n$

$$\lambda = \left(\frac{K\ell}{r_s\pi}\right)^2 \frac{F_y}{E}$$

K = 1.5 (≈ avg. about weak and strong axis pile)

ℓ = 80.4 in. (about weak axis, would be 52.8 in. about strong taking depth-of-fixity based upon moment)

F_y = 50 ksi

E = 29000 ksi

r_s = 2.41 in. (about weak axis)

A_s = 12.4 in.^{2}

$$\lambda = \left(\frac{1.5 \times 80.4}{2.41 \times \pi}\right)^2 \frac{50}{29000} = 0.44$$

$\lambda \leq 2.25$,

$\therefore P_r = (1.0)0.66^{\lambda}F_yA_s = (1.0)\,0.66^{0.44} \times 50 \times 12.4 =$ 516.4 kips

And,

$$\frac{P_u}{P_r} = \frac{41.7}{516.4} = 0.08 < 0.2,$$

$$\therefore \frac{P_u}{2P_r} + \left(\frac{M_{ux}}{M_{rx}} + \frac{M_{uy}}{M_{ry}}\right) = \frac{41.7}{2 \times 516.4} + \left(\frac{125.6}{180.8} + \frac{9.1}{59.2}\right) = 0.89 \leq 1.0 \text{ OK}$$

9.b. Load Case 2 – Transverse Dominant

For Load Case 2 (HP 10 x 42),

$M_{ux} = M_y^D$ = 37.7 k-ft.

$M_{uy} = M_z^D$ = 30.4 k-ft.

$M_{rx} = F_yS_x = 50\,\text{ksi} \times 43.4\,\text{in.}^3 \times {}^{1\,\text{ft}}/_{12\,\text{in.}}$ = 180.8 k-ft.

$M_{ry} = F_yS_y = 50\,\text{ksi} \times 14.2\,\text{in.}^3 \times {}^{1\,\text{ft}}/_{12\,\text{in.}}$ = 59.2 k-ft.

$P_u = P^D$ (max.) = 57.4 kips

$P_r = \phi P_n = 1.0P_n$

$$\lambda = \left(\frac{K\ell}{r_s\pi}\right)^2 \frac{F_y}{E}$$

K = 1.5 (≈ avg. about weak and strong axis pile)

ℓ = 80.4 in. (about weak axis, would be 52.8 in. about strong taking depth-of-fixity based upon moment)

F_y = 50 ksi

E = 29000 ksi

r_s = 2.41 in. (about weak axis)

A_s = 12.4 in.^{2}

$$\lambda = \left(\frac{1.5 \times 80.4}{2.41 \times \pi}\right)^2 \frac{50}{29000} = 0.44$$

$\lambda \le 2.25$,

$\therefore P_r = (1.0)0.66^{\lambda}F_yA_s = (1.0)\,0.66^{0.44} \times 50 \times 12.4 =$ 516.4 kips

And,

$$\frac{P_u}{P_r} = \frac{57.4}{516.4} = 0.11 < 0.2\,,$$

$$\therefore\ \frac{P_u}{2P_r} + \left(\frac{M_{ux}}{M_{rx}} + \frac{M_{uy}}{M_{ry}}\right) = \frac{57.4}{2 \times 516.4} + \left(\frac{37.7}{180.8} + \frac{30.4}{59.2}\right) = 0.78 \le 1.0\ \text{OK}$$

10. Discussion of Flexible Versus Standard Design Options

The flexible option for seismic design permits the designer to choose piles (number of, and sizes) which may not be significantly increased over non-seismic design considerations. This is true for the bridge in the current example. Overstrength plastic behavior is relied upon in the substructures and foundations more heavily than for most bridges on the State System through increased R-factors (and smaller design moments). The chances for a span or spans to be lost during a significant seismic event are probably not significantly increased when a bridge is designed according to the flexible option. However, the amount of allowable (and probably costly) damage to a bridge is increased.

If the piles in the current example were checked using design moments computed from the higher R-factors recommended for most bridges on the State System, they would have been found not adequate. At the piers, the combined axial force and bi-axial bending ratio would be about 1.24 and at the abutments it would be about 1.31. Both of these values are significantly greater than 1.0. As a consequence, larger size piles, at a minimum, would be required if the example bridge was to be built on the State System.

It should be noted, however, that there can be what could be termed a “double jeopardy” associated with increasing piles sizes (and/or numbers of piles). This effect is illustrated well in the design calculations for the bridge in the current example. If the piles sizes were increased, the substructures and foundations (i.e. the bridge in critical elements) would also become stiffer. This in turn would cause the longitudinal and transverse periods to become shorter. Since the periods of the more flexible structure are well past the upper plateau of the design response spectrum, a less flexible structure will cause the total design base shears in both directions (longitudinal and transverse) to increase from the levels calculated above. At some point, the decreased periods and resulting increased seismic design loads will converge with the required increases in pile sizes and/or numbers. In some cases, this may not happen until one or both of the bridge periods (probably transverse) reaches the plateau (or flat part) of the design response spectrum.

11. Pile Shear Structural Capacity Check, and Pile Connection Details, and Cap Reinforcement Details

Shear capacity of the piles at the piers and abutments should be checked, but is generally not a concern in steel HP piles. For most bridges in Illinois, a pile embedment of 2 ft. – 0 in. is optimally specified for piles into pier and abutment caps such that plastic moments in the piles can be developed for accelerations which approach the design seismic event. However, primarily due to cost concerns, it is desirable to limit cap depths to about 2 ft. – 6 in. for PPC deck beam bridges. When cap depths are limited, special anchorage details should be provided at the ends of piles which effectively extends the embedment depth. At pier caps, some nominal added confinement reinforcement should also be provided to help ensure that plastic moments in the piles will develop.

11.a. Shear Capacity Check of HP Piles

LRFD Article 6.10.9 should be used to check the shear capacity of the piles at the piers and abutments. The following LRFD Code equations apply when determining shear capacity for seismic loadings. Basic shear capacity is calculated as,

$$V_u \leq \phi V_n \quad \text{(Eq. 6.10.9.1-1)}$$

V_n is determined from:

$$V_n = CV_p \quad \text{(Eq. 6.10.9.2-1)}$$

In which:

$$V_p = 0.58F_yDt \quad \text{(Eq. 6.10.9.2-2)}$$

Where:

ϕ = 1.0

D = depth of member or width of flanges (in.)

t = thickness of web or twice the thickness of the flanges (in.)

F_y = yield strength of the pile (ksi)

C = ratio of shear buckling strength to shear yield strength taken as 1 for HP piles subjected to extreme event loadings

Capacity of HP 10 x 42 piles for shear:

Longitudinal Direction (Strong Axis),

ϕV_n = $(1.0)(1.0)0.58F_yDt$

D = 9.7 in.

t = 0.415 in

F_y = 50 ksi

ϕV_n = $(1.0)(1.0)0.58F_yDt$ = 0.58(50)(9.7)0.415) = 116.7 kips

Transverse Direction (Weak Axis),

ϕV_n = $(1.0)(1.0)0.58F_yDt$

D = 10.08 in.

t = 0.84 in

F_y = 50 ksi

ϕV_n = $(1.0)(1.0)0.58F_yDt$ = 0.58(50)(10.08)0.84) = 245.5 kips

Design Shears for 10 x 42:

V_u (Load Case 1 – Longitudinal Dominant)

V_z^D = 27.3 kips

V_y^D = 4.1 kips

V_u (Load Case 2 – Transverse Dominant)

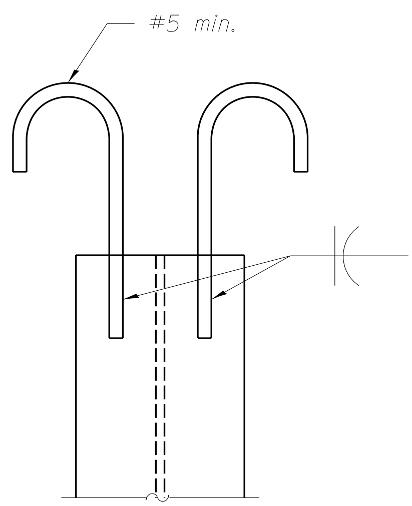
V_z^D = 8.2 kips

V_y^D = 13.6 kips

For 10 x 42, $V_u << \phi V_n$ OK and HP 12 x 53 OK by inspection.

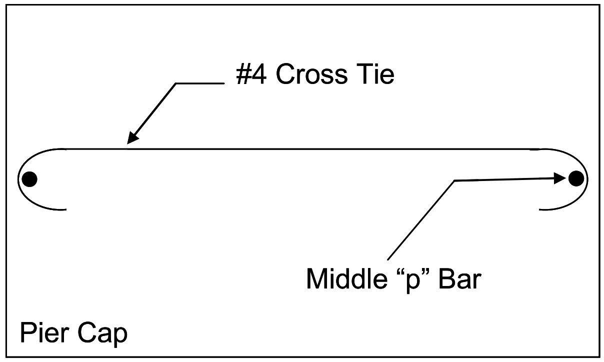
11.b. Anchorage Details at Piers and Abutments for HP Piles

The detail shown below or one of the other two options shown in Figure 3.15.5.5-1 of the Bridge Manual may be used at the ends of piles which are only embedded 1 ft. – 0 in. into pier and abutment caps. Using 4 - #5 bars (F_y = 60 ksi) in which the tops of the 180° hooks extend to 1 ft. – 0 in. above the end of the pile provides about 74 kips of nominal tensile capacity. This is more that adequate to resist pile pullout and also provides equivalent added embedment.



11.c. Added Pier Cap Confinement Reinforcement

In addition to the normal closed stirrups ("s" bars), the longitudinal reinforcement ("p" bars along cap transverse to bridge) and the end reinforcement ("u" bars); cross ties (#4's) which have 180° hooks at each end should be provided on the middle "p" bars at 2 ft. – 0 in. cts. minimum for added confinement. A simple sketch showing these bars is shown below.



12. Dowel Bar Connection of Beams to Pier and Abutment Caps

As discussed in the introduction to the current example bridge, the connection between PPC deck beams and substructures has historically been strong or not fuse like which is not optimal according to the philosophy of IDOT’s ERS strategy. Two 1 in. ϕ dowel rods have traditionally been used for the connection. It is desirable to make these connections somewhat weaker (more fuse like) without potentially compromising the integrity of these connections for other structural and service conditions these bridges may experience during their lifetime. As such, the two dowel rods at each end of a beam (or an equivalent) should be designed to resist a shear force equal to the lesser of 0.4 or C_{sm} (the design acceleration coefficient) times the tributary dead weight of beams plus overlay. For other classes of bridges in Illinois, the fraction of dead weight used for this design load is 0.2. In addition, connection rods should not be less than ¾ in. in diameter with a minimum yield strength of 36 ksi (tensile strength of 58 ksi). Note that these provisions also apply for the design of the connections in single span PPC deck beam bridges.

The shear strength of connection rods (or bars) should be calculated according to the equation below with a ϕ factor of 1.0. The same equation for shear strength is used in Section 3.7.3 of the Bridge Manual but with a ϕ factor of 0.75. It is from LRFD Article 6.13.2.12.

$$\phi R_n = \phi 0.48 A_b F_u \qquad \text{(Eq. 6.13.2.12-1)}$$

Where:

ϕ = 1.0

A_b = area of dowel rod (in.^{2})

F_u = tensile strength of rod (ksi)

Maximum Design Shear Force for Anchor Rods:

Use 0.39 (< 0.4) times tributary dead weight flowing into ends of 60 ft. span beams as the governing design shear.

$$R_u = 0.39 \times \frac{\left(\frac{1}{2} \times 60\text{ ft.}\right)\left(0.864\text{ k/ft.} + 4\text{ ft.} \times 0.05\text{ ksf}\right)}{2\text{ rods}} = 6.2\text{ kips/rod}$$

Allowable Shear per Rod:

Compute strengths of ¾ in. ϕ Grade 36 and Grade 55 rods.

$\phi R_n = \phi 0.48 A_b F_u$

ϕ = 1.0

A_b = 0.44 in.^{2}

F_u = 58 and 75 ksi

$\phi R_n = \phi 0.48 A_b F_u =$ (1.0)(0.48)(0.44)(58) = 12.2 kips (Grade 36)

$\phi R_n = \phi 0.48 A_b F_u =$ (1.0)(0.48)(0.44)(75) = 15.8 kips (Grade 55)

Therefore, ¾ in. ϕ dowels of Grade 36 or Grade 55 are adequate (#6 rebar, F_y = 60 ksi, may also be substituted).

13. Minimum Support Length (Seat Width) Requirements at Piers and Abutments

At a minimum, adequate seat widths shall be provided in the longitudinal direction only at piers and abutments according to LRFD Article 4.7.4.4. The requirements of the LRFD Code are generally not as stringent as those detailed in Section 3.15.4.2 of the Bridge Manual, and are permitted for single and multi-span simply supported PPC deck beam bridges designed according to the flexible option outlined in Section 3.15.8 of the Bridge Manual.

For bridges in Zone 2, the minimum required seat width at piers and abutments shall be calculated as 150% of the following equation from the LRFD Code.

$$N = (8 + 0.02L + 0.08H)(1 + 0.000125S^2) \text{ in.} \qquad \text{(Eq. 4.7.4.4-1)}$$

Where:

L	=	total length of bridge for single or multi-span PPC deck beam bridges (ft.)
H	=	average height of piers for multi-span PPC deck beam bridges and taken a zero (0.0) for single span bridges (ft.)
S	=	bridge skew (°)

L is taken as the total length of the bridge due to the close proximity of the beam ends at piers (i.e. they will easily push against each other if the anchor rods fuse). Since PPC deck beams are already “wide” (3 ft. or 4 ft.), the transverse seat width requirements have been waived for these bridges (but they are required, over and above that specified by the LRFD Code, for most structures on the State System as outlined in Section 3.15.4.2 of the Bridge Manaul).

Minimum Required Support Length:

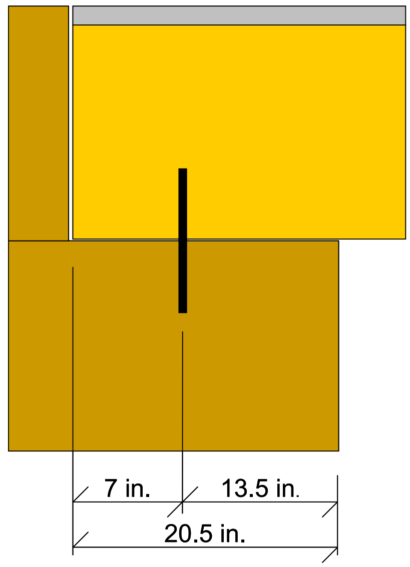
$$N = (8 + 0.02L + 0.08H)(1 + 0.000125S^2) \text{ in.}$$

L	=	140 ft.
H	=	10 ft.
S	=	0°

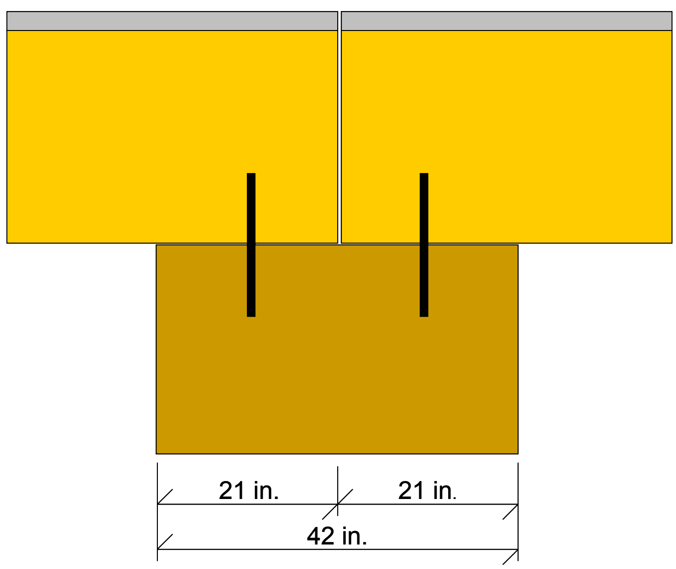
$$\therefore\ N = (8 + 0.02 \times 140 + 0.08 \times 10)(1 + 0.000125 \times 0^2) = 11.6 \text{ in.}$$

$$\text{and } 1.5N = 17.4 \text{ in. Min. Req. Support Length}$$

Provided Support Length at Abutments:



Provided Support Length at Piers:



Therefore, provided support lengths at abutments and pier are adequate.

20.5 in. > 17.4 in. and 21 in. > 17.4 in. OK

14. Overview of Example Bridge Design With Metal Shell Piles

If the piles for the current example bridge were metal shell (MS) and not HP type; the design steps, procedures and calculations would be quite similar to those presented in Parts 1 through 13 above. However, there are also some important differences. Given below is an abbreviated design for the current example bridge with metal shell piles. Emphasized are the differences in design procedures, etc. when metal shell piles are employed instead of HP piles.

14.a. Determination of Bridge Periods and Base Shears – Transverse and Longitudinal Directions

The MS piles to be used for the example bridge are 14 in. x 0.25 in. (F_y = 45 ksi, f'_c = 3.5 ksi assumed for design). As shown in the calculations above, the HP piles did not significantly contribute to the “seismic weight” of the bridge. It is assumed here that the MS piles do not either. As such, 1220 kips will again be used as the mass (weight) of the bridge for seismic design.

The transverse and longitudinal periods of the example bridge with MS piles can be calculated in nearly the same manner as when the piles are HP. The primary difference is in the stiffness calculation for the piles (and primarily involves the moment of inertia and/or the modulus of elasticity). Other than this, Parts 1.b. through 1.f. for determining the transverse period, etc., Parts 2.b. through 2.d. for determining the longitudinal period, etc., and the methods for determining and distributing total seismic base design shears to the abutments and piers in Part 3 are the same as outlined above.

It should be assumed that MS piles are reinforced concrete columns in soil with the shell acting as both the longitudinal (vertical) and transverse (“spiral”) reinforcement. MS piles may also have added spirals and longitudinal bars, but this scenario is not considered in the current example. For typical or regular bridges, it is recommended that the stiffness of the piles at the piers and abutments should be determined using the “equivalent” moments of inertia, I_p, given in Appendix C for MS piles (even if there are added longitudinal bars considered) and the modulus of elasticity for steel, E_s (29000 ksi), as shown in the basic equation for pier and abutment stiffnesses below.

$$k_{Pier\ or\ Abut} = \frac{(\text{no. of piles})(12\ \text{or}\ 3) \times E_s \times I_p}{h_p^3}\text{; where } h_p = \text{pile height}$$

Alternatively, the pile moment of inertia may be computed based upon the steel only with the equation below. In this case, E_s should be substituted with E_e (an equivalent modulus of elasticity using Eq. 6.9.5.1-5 of the LRFD Code), and I_p should be substituted with I_{ps} in the equation above.

$$I_{ps} = \frac{\pi\left(\phi_{pile}/2\right)^4}{4} - \frac{\pi\left(\phi_{pile}/2 - t_w\right)^4}{4}\text{; where } \phi_{pile} = \text{gross pile diameter, and}$$

$$t_w = \text{thickness of pile shell}$$

The values for h_p are determined in the same manner as for HP piles (see also Appendix C). The fixed-fixed case should be used for the transverse direction (with the 12 factor) and the fixed-pinned case should be used for the longitudinal direction (with the 3 factor). In the longitudinal direction, the “softening” effects due to rigid body rotation of caps at piers and abutments should be taken account of in the same manner as for HP piles.

Using depths-of-fixity from Appendix C (fixed-fixed depth = 9.1 ft., and fixed-pinned depth = 2 x 4.5 ft = 9.0 ft.), a modulus of elasticity of steel equal to 29000 ksi, and an equivalent moment of inertia of 358.4 in.^{4}, the following pier and abutment stiffnesses were obtained:

$k_{Pier(Trans)}$ = 146.7 k/in.

$k_{Abut(Trans)}$ = 478.9 k/in.

$k_{Pier(Long)}$ = 29.9 k/in.

$k_{Abut(Long)}$ = 87.4 k/in.

The transverse period was calculated to be 0.55 sec. using the simplified method and the longitudinal period was determined to be 0.73 sec. As can be observed, the example bridge with metal shell piles is stiffer in the transverse and longitudinal directions than when the piles were HP. As such, the total base shears in the transverse and longitudinal directions will be greater for the current example structure when MS piles are used. The following are the computed design base shears at the piers and abutments for the transverse and longitudinal directions:

$V_{Base\ Shear\ P(T)}$ = 239.4 kips
$V_{Base\ Shear\ A(T)}$ = 95.8 kips
$V_{Base\ Shear\ P(L)}$ = 63.8 kips
$V_{Base\ Shear\ A(L)}$ = 186.3 kips

14.b. Frame Analysis and Seismic Design Forces for Piers and Abutments

The procedures outlined above for calculation of forces from overturning and frame action are identical irregardless of whether the piles are HP or metal shell. The methods for employing R-factors, P-Δ effects and combination of orthogonal forces are also identical. However, since the piles have a round cross-section, the final seismic design shears and moments for each load case may be further simplified by performing a vector addition of the transverse and longitudinal directions (See Example 1 Part 6.c.). Given below are the orthogonally combined seismic loadings for the piles in the piers and abutments along with the further simplified vector sums of the shears and moments.

Piers - Load Case 1 - Longitudinal Dominant (per pile):

$$V_z^D = 9.1 \text{ kips}$$

$$V_y^D = 10.3 \text{ kips}$$

$$M_z^D = 23.2 \text{ k-ft.}$$

$$M_y^D = 35.5 \text{ k-ft.}$$

$$P^D = 22.4 \text{ and } 102.0 \text{ kips}$$

Vector addition of the shears and moments further simplifies the design loads.

$$V^D = \sqrt{9.1^2 + 10.3^2} = 13.7 \text{ kips}$$

$$M^D = \sqrt{23.2^2 + 35.5^2} = 42.4 \text{ k-ft.}$$

$$P^D = 22.4 \text{ and } 102.0 \text{ kips}$$

Piers - Load Case 2 - Transverse Dominant (per pile):

$V_z^D = 2.7 \text{ kips}$

$V_y^D = 34.2 \text{ kips}$

$M_z^D = 77.5 \text{ k-ft.}$

$M_y^D = 10.6 \text{ k-ft.}$

$P^D = -70.4 \text{ (Ten.) and } 194.8 \text{ kips}$

Vector addition of the shears and moments further simplifies the design loads.

$V^D = \sqrt{2.7^2 + 34.2^2} = 34.3 \text{ kips}$

$M^D = \sqrt{77.5^2 + 10.6^2} = 78.2 \text{ k-ft.}$

$P^D = -70.4 \text{ (Ten.) and } 194.8 \text{ kips}$

Abutments - Load Case 1 - Longitudinal Dominant (per pile):

$V_z^D = 37.3 \text{ kips}$

$V_y^D = 5.8 \text{ kips}$

$M_z^D = 17.5 \text{ k-ft.}$

$M_y^D = 174.1 \text{ k-ft.}$

$P^D = 23.5 \text{ and } 46.3 \text{ kips}$

Vector addition of the shears and moments further simplifies the design loads.

$V^D = \sqrt{37.3^2 + 5.8^2} = 37.7 \text{ kips}$

$M^D = \sqrt{17.5^2 + 174.1^2} = 175.0 \text{ k-ft.}$

$P^D = 23.5 \text{ and } 46.3 \text{ kips}$

Abutments - Load Case 2 - Transverse Dominant (per pile):

$V_z^D = 11.2$ kips

$V_y^D = 19.2$ kips

$M_z^D = 58.3$ k - ft.

$M_y^D = 52.2$ k - ft.

$P^D = -3.1$ (Ten.) and 72.9 kips

Vector addition of the shears and moments further simplifies the design loads.

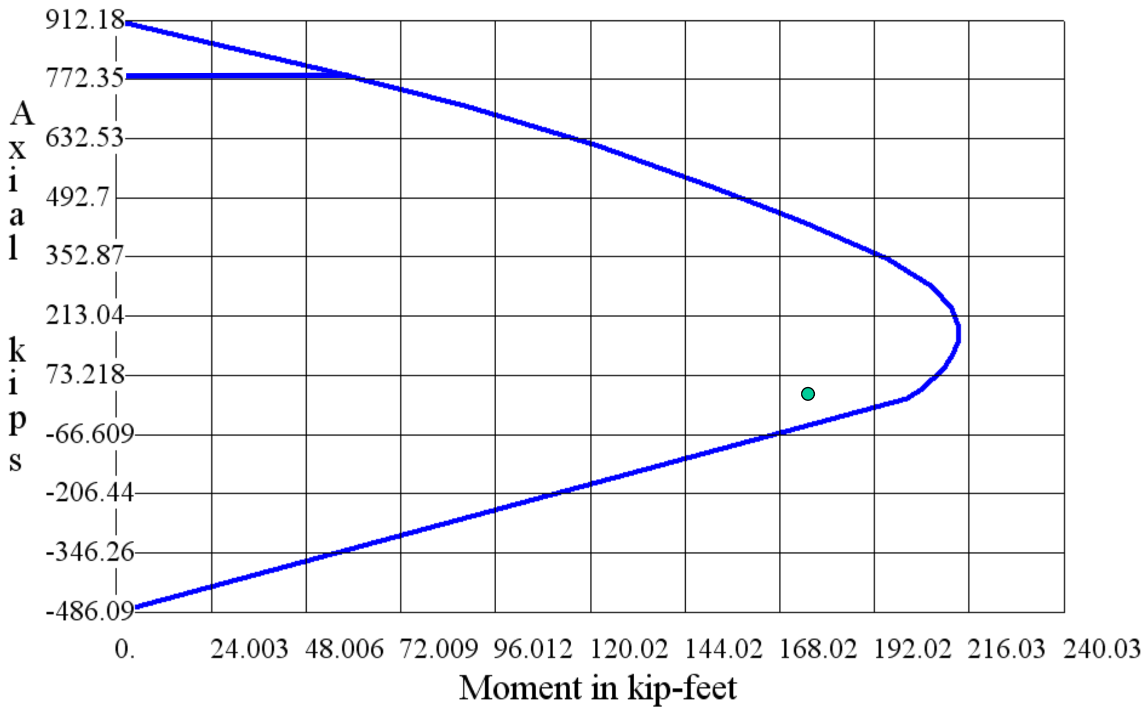
$V^D = \sqrt{11.2^2 + 19.2^2} = 22.2$ kips

$M^D = \sqrt{58.3^2 + 52.2^2} = 78.3$ k - ft.

$P^D = -3.1$ (Ten.) and 72.9 kips

14.c. Combined Axial Force and Bending Structural Capacity Check for Piers and Abutments

Appendix C provides nominal axial force - moment interaction strength diagrams for metal shell piles which do not have added vertical or spiral reinforcement. The diagrams use a ϕ factor of 1.0 as permitted in the IDOT ERS strategy and are suitable for design. They were developed using standard methods of structural engineering with the metal shell considered the columnar reinforcement according to LRFD Article 5.13.4.5.2. The axial force-moment interaction diagram for MS 14 in. x 0.25 in. piles is shown below. The critical load case for both the piers and abutments (Abutments – Load Case 1) is also superimposed on the diagram. As can be observed, the loading is within the permissible envelope.



14.d. Pile Shear Structural Capacity Check, Minimum Steel, and Pile Connection Details

Seismic provisions for metal shell piles in Zone 2 are outlined in LRFD Article 5.13.4.6.2. Provisions for Zones 3 and 4 are given in LRFD Article 5.13.4.6.3. As indicated in Part 14.c. above, the metal shell is permitted to be considered as reinforcement. For many situations in Zone 2 (and Zone 1), added reinforcement bars in the piles may not be required for multi-span (or single span) simply supported PPC deck beam bridges designed using the flexible option except for anchorage bars which are required when the piles are not embedded a full 2 ft. – 0 in. into caps at piers and abutments.

The cross sectional area of steel for 14 in. x 0.25 in. metal shells used in the current example is 10.8 in.^{2}. This is equivalent to almost 35 - #5 bars. So, the "reinforcement" provided by these piles is substantial. The vertical steel ratio, ρ, based upon the gross area of the piles (14 in. ϕ) is,

$$\rho = \frac{10.8\text{ in.}^2}{\pi \times 7^2\text{ in.}^2} = 0.0702$$

LRFD Article 5.13.4.6.2a requires pile anchorage to have a minimum steel ratio of 0.01 which equates to,

$$A_a = 0.01\left(\pi \times 7^2 \text{ in.}^2\right) = 1.54 \text{ in.}^2$$

or about 5 - #5 bars. A modified version of the detail for metal shell reinforcement at abutments given on Departmental Base Sheet F-MS may be used as an anchorage detail when pile embedment at piers or abutments is only 1 ft. – 0 in. The detail calls for 6 - # 5 bars which are hooked 90° just above the end of the pile. The Base Sheet detail should be modified using the detail shown in Part 11.b. above (or Bridge Manual Figure 3.15.5.5-1 Option B) as a guide. The bars should extend 1 ft. – 0 in. above the pile and be hooked 180°. When metal shell piles are embedded 2 ft. – 0 in., no added anchorage (connection) reinforcement is typically needed except as required by design. See also Appendix C and Section 3.15.5.5 of the Bridge Manual for further guidance. By inspection, uplift (tension) is not a concern for the bridge in the current example (the nominal capacity of 6 - #5 bars is ≈ 110 kips).

Minimum vertical (longitudinal) and spiral reinforcement requirements are outlined in LRFD Article 5.13.4.6.2b. The minimum vertical steel reinforcement ratio is 0.005 which is satisfied by inspection from the calculation of ρ above (0.0702). The spiral reinforcement ratio provided by the metal shell may be calculated as follows,

$$\rho_{s(\text{Provided})} = \frac{\text{Volume of steel in a 1 in. height of shell}}{\text{Volume of concrete in a 1 in. height of shell}} = \frac{10.8 \text{ in.}^3}{1 \times \pi \times 6.75^2 \text{ in.}^3} = 0.0755$$

Article 5.13.4.6.2b requires a #3 spiral at 4 in. within the upper plastic hinging region which equates to a spiral reinforcement ratio of (with the “spiral diameter” equal to 14 in. – 0.25 in. = 13.75 in.),

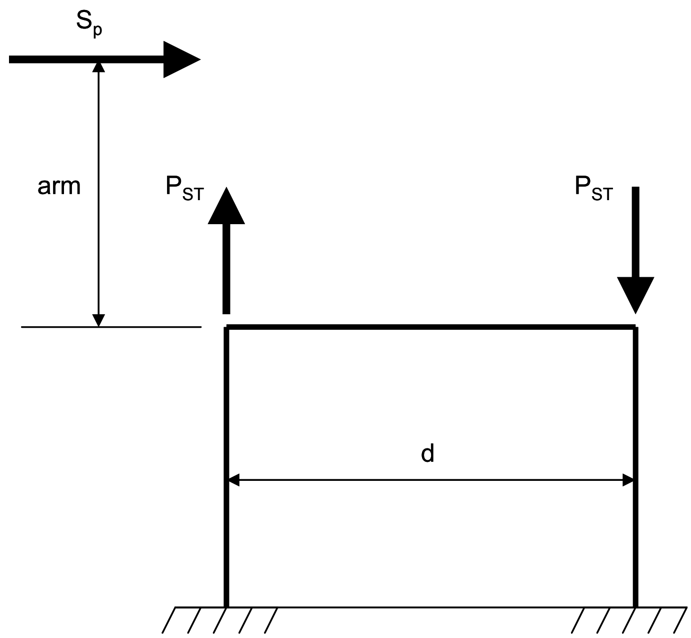
$$\rho_s = \frac{\text{Volume of 1 spiral turn}}{\text{Volume of concrete in 1 spiral turn}} = \frac{0.11 \times \pi \times 13.75}{\pi \times 6.75^2 \times 4} = 0.0083$$

Therefore, the shell provides just over 9 times the minimum required steel and is adequate for confinement. A cursory check of the shear strength provided by the shell (see the method presented in Example 1 Part 7.b.) in comparison to the design shears also shows ample adequacy.

14.e. Pier Cap Reinforcement, Connection of Beams to Pier and Abutment Caps, and Support Lengths

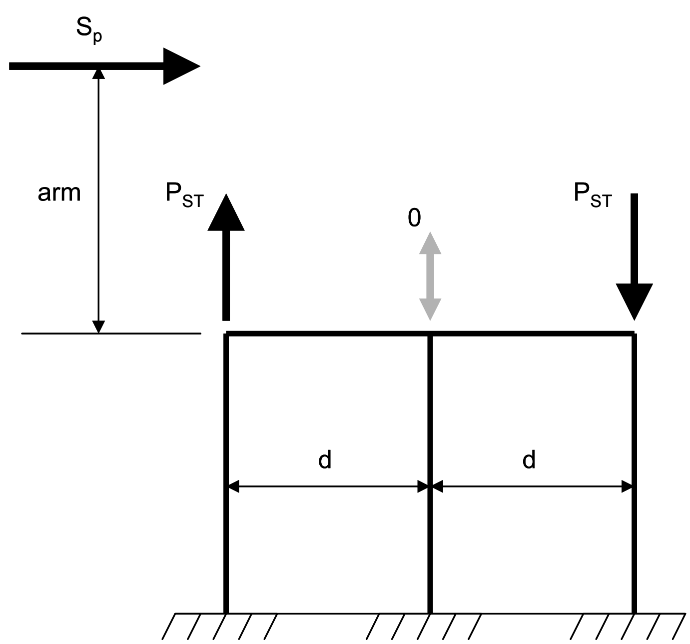
Pier caps should have the same added reinforcement described in Part 11.c above. The dowel bar connection should be designed according to Part 12, and support lengths should be verified according to Part 13.

Appendix A: Columnar Axial Force Solutions due to Overturning Moment for 12 Cases



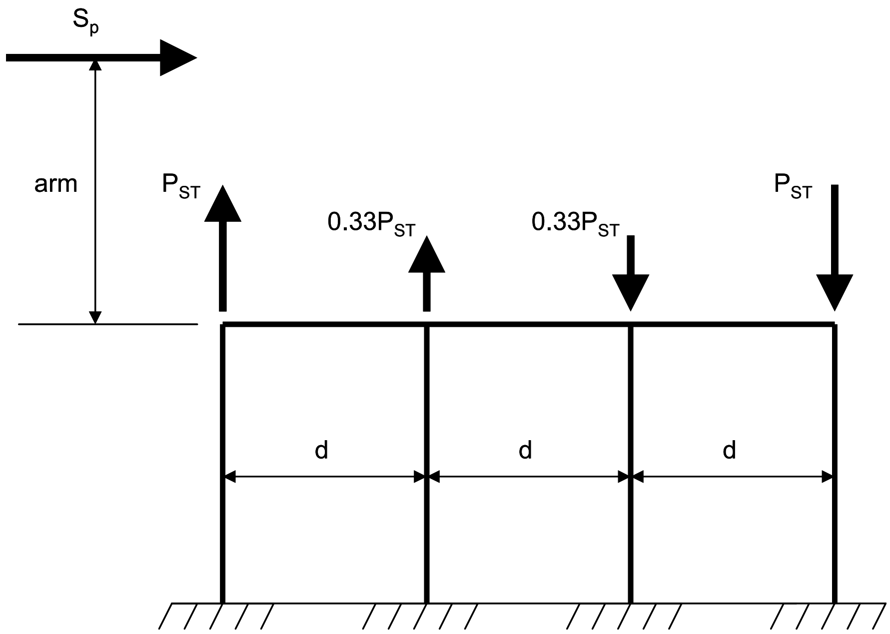
$$M = S_p \times arm$$
$$M = P_{ST} \times d$$
$$\therefore P_{ST} = M/d$$

Case A.1: Overturning Axial Forces for 2 Column Piers or 2 Piles in a Row



$$M = S_p \times arm$$
$$M = P_{ST} \times 2d$$
$$\therefore P_{ST} = M/2d$$

Case A.2: Overturning Axial Forces for 3 Column Piers or 3 Piles in a Row

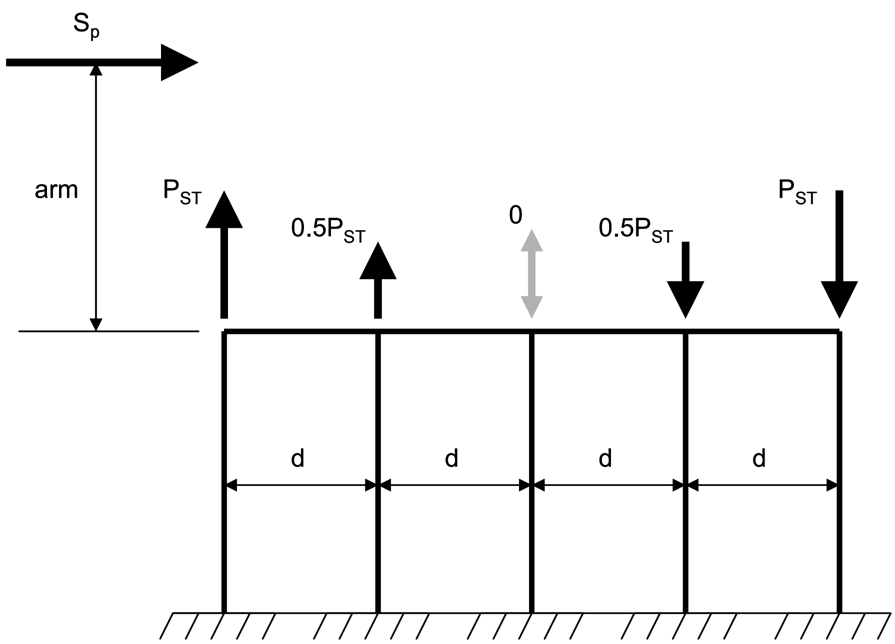


$$M = S_p \times arm$$

$$M = 2\left(P_{ST} \times \tfrac{3}{2} d\right) + 2\left(\tfrac{1}{3} P_{ST} \times \tfrac{1}{2} d\right)$$

$$\therefore P_{ST} = \frac{3}{10} M/d$$

Case A.3: Overturning Axial Forces for 4 Column Piers or 4 Piles in a Row

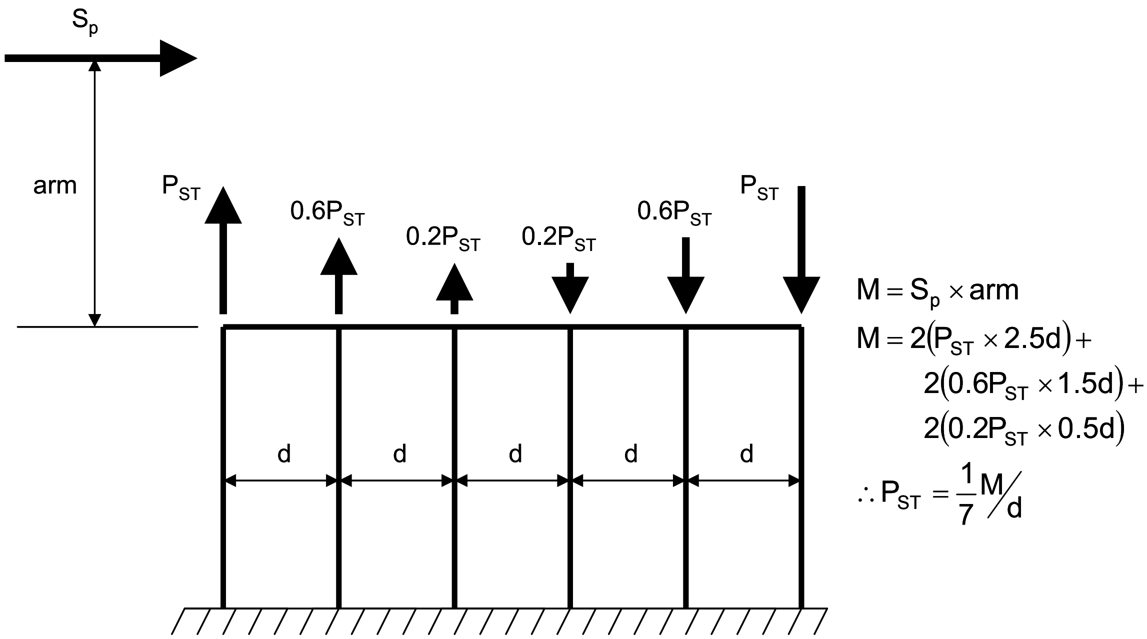


$$M = S_p \times arm$$

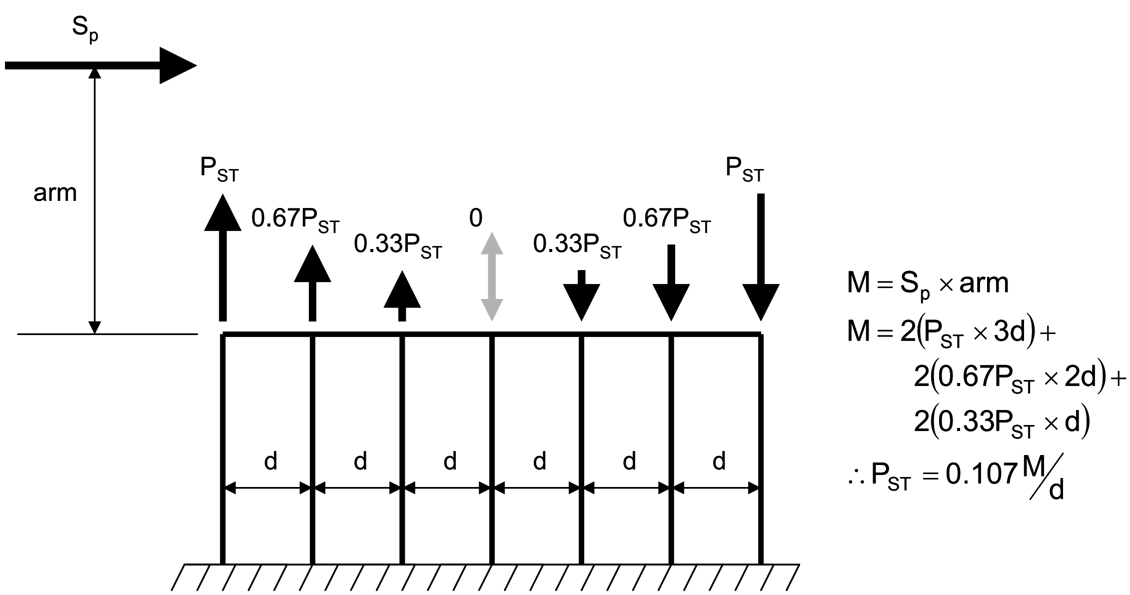
$$M = 2\left(P_{ST} \times 2d\right) + 2\left(\tfrac{1}{2} P_{ST} \times d\right)$$

$$\therefore P_{ST} = \frac{1}{5} M/d$$

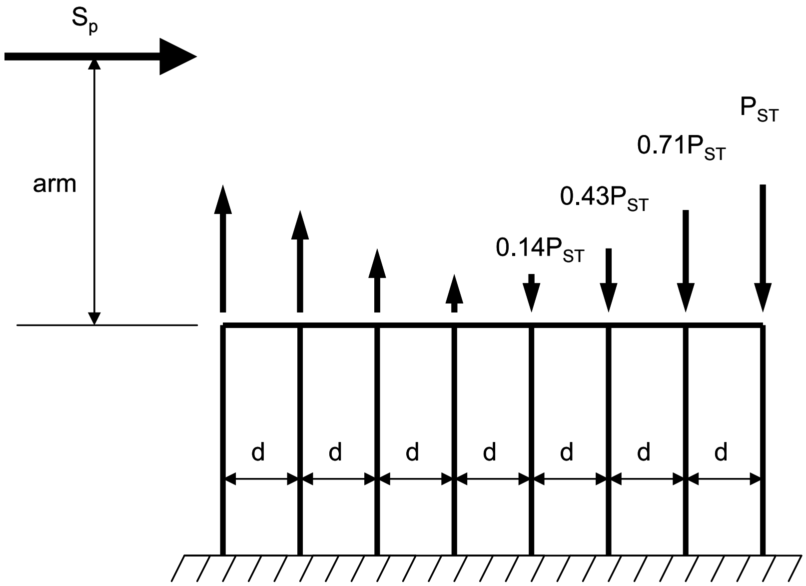
Case A.4: Overturning Axial Forces for 5 Column Piers or 5 Piles in a Row



Case A.5: Overturning Axial Forces for 6 Column Piers or 6 Piles in a Row

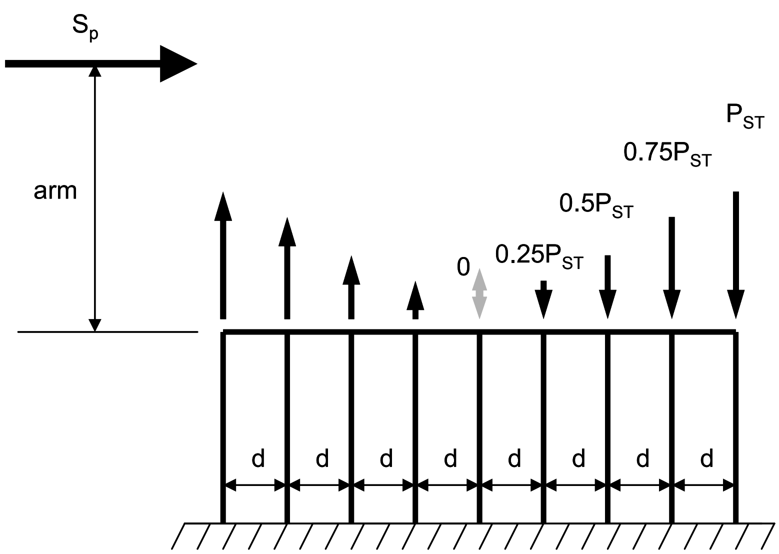


Case A.6: Overturning Axial Forces for 7 Column Piers or 7 Piles in a Row



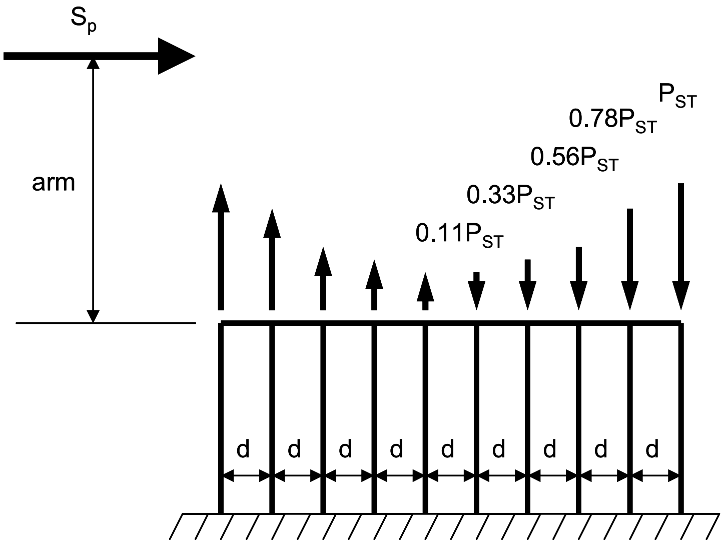
$$M = S_p \times arm$$
$$M = 2(P_{ST} \times 3.5d) + 2(0.71P_{ST} \times 2.5d) + 2(0.43P_{ST} \times 1.5d) + 2(0.14P_{ST} \times 0.5d)$$
$$\therefore P_{ST} = \frac{1}{12} M/d = 0.083 M/d$$

Case A.7: Overturning Axial Forces for 8 Column Piers or 8 Piles in a Row



$$M = S_p \times arm$$
$$M = 2(P_{ST} \times 4d) + 2(0.75P_{ST} \times 3d) + 2(0.5P_{ST} \times 2d) + 2(0.25P_{ST} \times d)$$
$$\therefore P_{ST} = \frac{1}{15} M/d = 0.067 M/d$$

Case A.8: Overturning Axial Forces for 9 Column Piers or 9 Piles in a Row

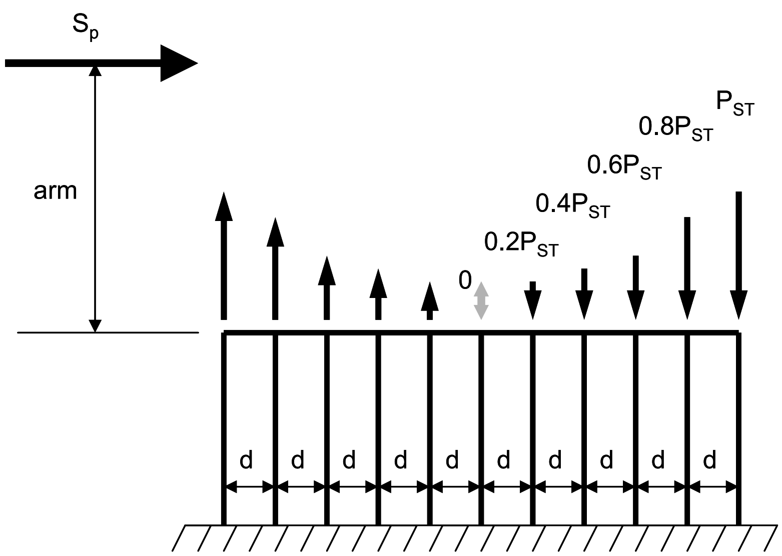


$$M = S_p \times arm$$

$$M = 2(P_{ST} \times 4.5d) + 2(0.78P_{ST} \times 3.5d) + 2(0.56P_{ST} \times 2.5d) + 2(0.33P_{ST} \times 1.5d) + 2(0.11P_{ST} \times 0.5d)$$

$$\therefore P_{ST} = 0.055\, M/d$$

Case A.9: Overturning Axial Forces for 10 Column Piers or 10 Piles in a Row

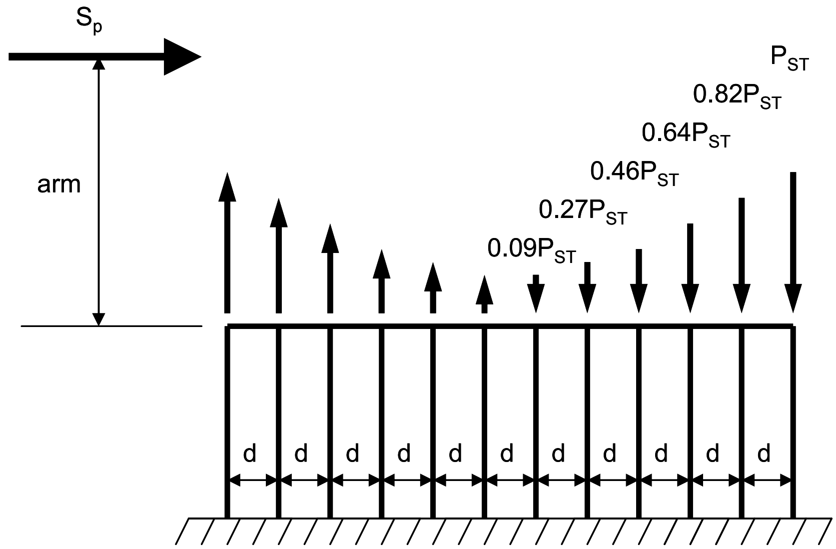


$$M = S_p \times arm$$

$$M = 2(P_{ST} \times 5d) + 2(0.8P_{ST} \times 4d) + 2(0.6P_{ST} \times 3d) + 2(0.4P_{ST} \times 2d) + 2(0.2P_{ST} \times d)$$

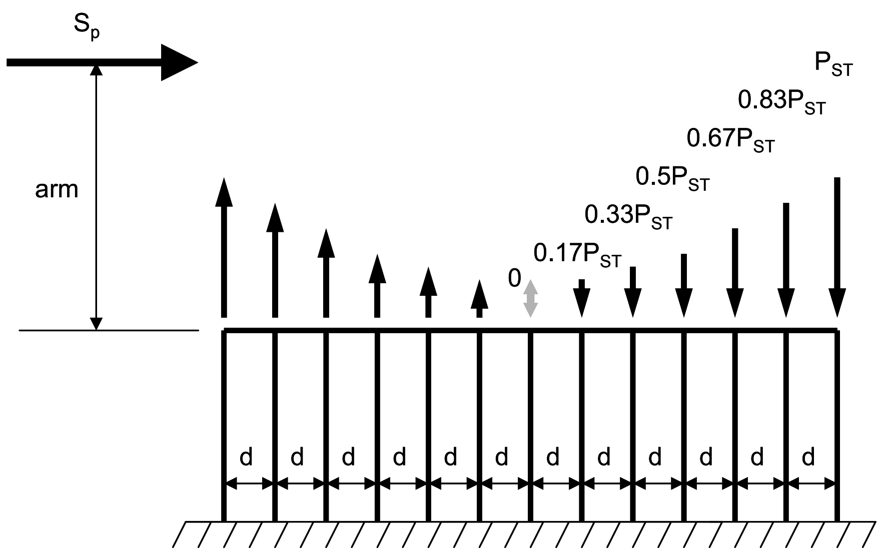
$$\therefore P_{ST} = \frac{1}{22}\, M/d = 0.045\, M/d$$

Case A.10: Overturning Axial Forces for 11 Column Piers or 11 Piles in a Row



$$M = S_p \times arm$$
$$M = 2(P_{ST} \times 5.5d) + 2(0.82P_{ST} \times 4.5d) + 2(0.64P_{ST} \times 3.5d) + 2(0.46P_{ST} \times 2.5d) + 2(0.27P_{ST} \times 1.5d) + 2(0.09P_{ST} \times 0.5d)$$
$$\therefore P_{ST} = \frac{1}{26} M/d = 0.038\, M/d$$

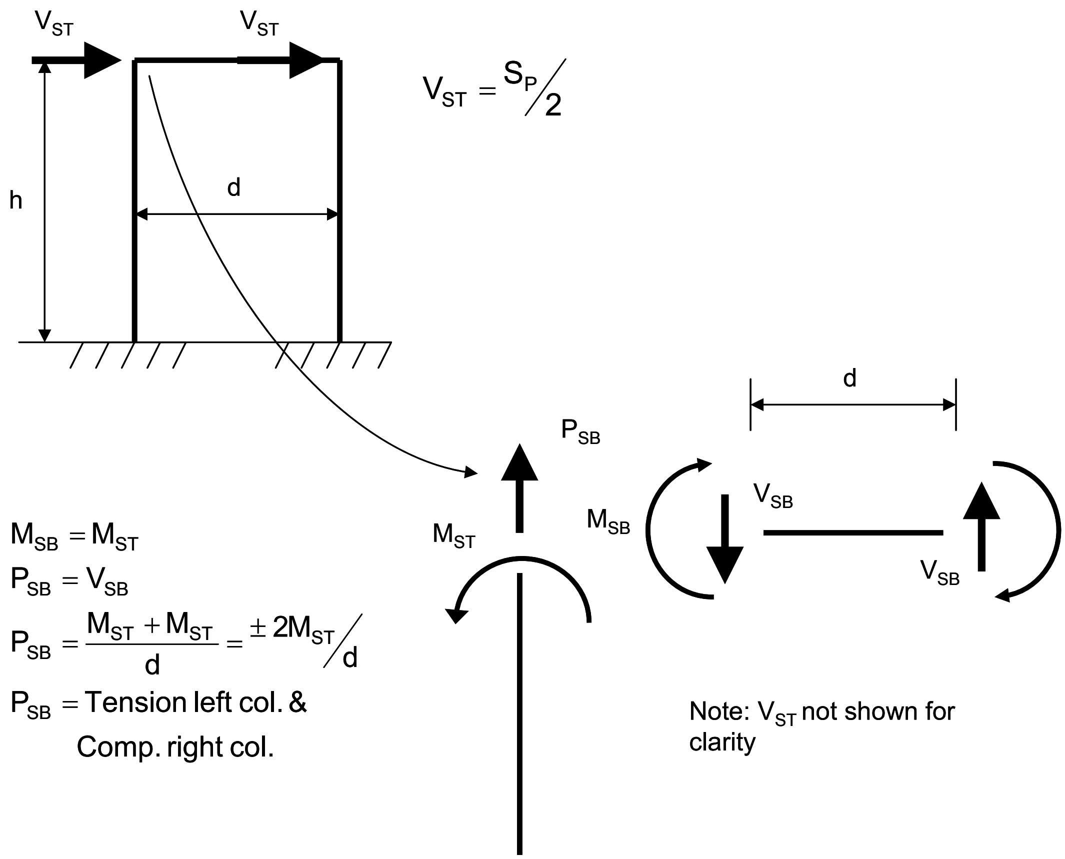
Case A.11: Overturning Axial Forces for 12 Column Piers or 12 Piles in a Row



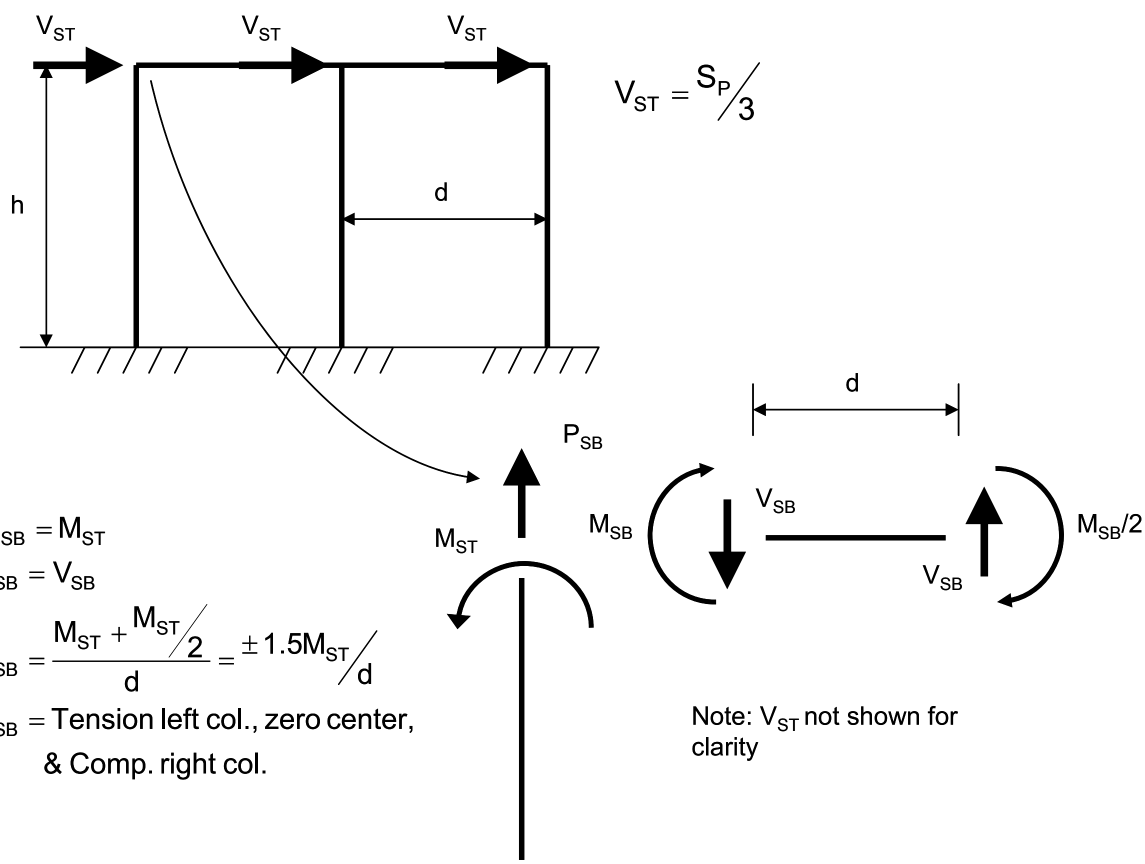
$$M = S_p \times arm$$
$$M = 2(P_{ST} \times 6d) + 2(0.83P_{ST} \times 5d) + 2(0.67P_{ST} \times 4d) + 2(0.5P_{ST} \times 3d) + 2(0.33P_{ST} \times 2d) + 2(0.17P_{ST} \times d)$$
$$\therefore P_{ST} = 0.033\, M/d$$

Case A.12: Overturning Axial Forces for 13 Column Piers or 13 Piles in a Row

Appendix B: Columnar Axial Force Solutions due to Frame Action for 5 Cases



Case B.1: Frame Action Axial Forces for 2 Column Piers or 2 Piles in a Row



$$V_{ST} = S_P/3$$

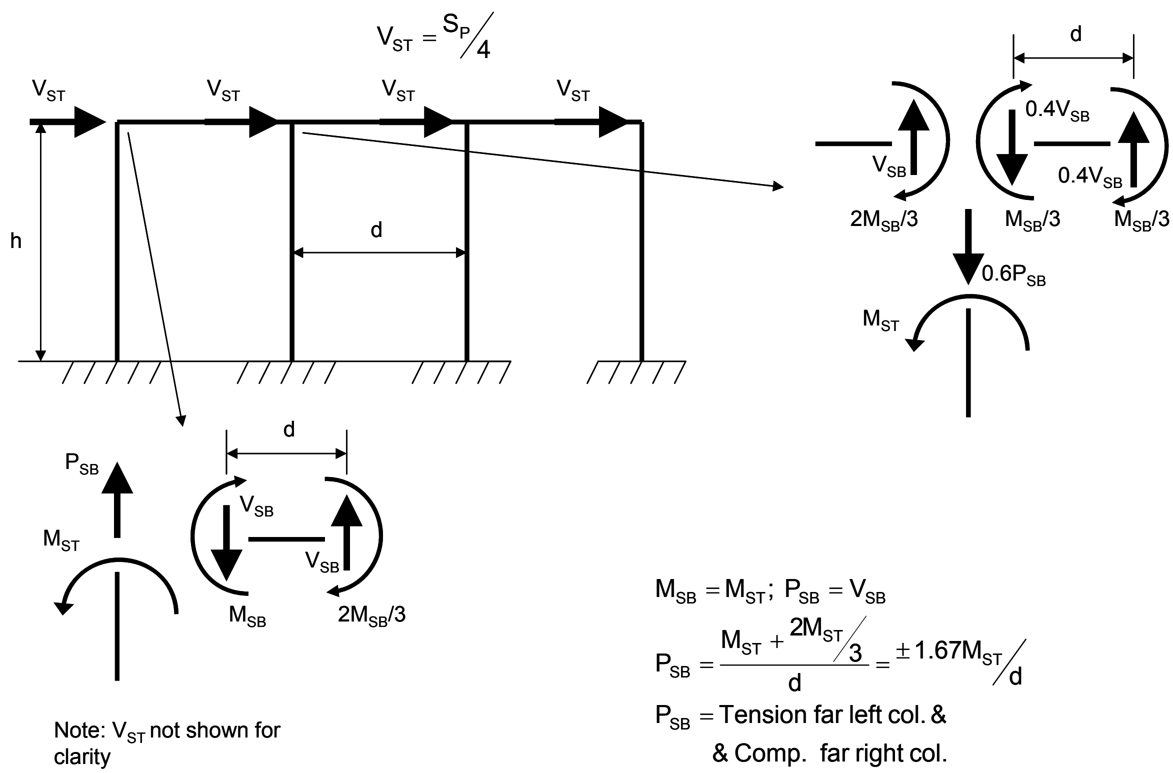
$$M_{SB} = M_{ST}$$

$$P_{SB} = V_{SB}$$

$$P_{SB} = \frac{M_{ST} + M_{ST}/2}{d} = \pm 1.5 M_{ST}/d$$

P_{SB} = Tension left col., zero center, & Comp. right col.

Case B.2: Frame Action Axial Forces for 3 Column Piers or 3 Piles in a Row

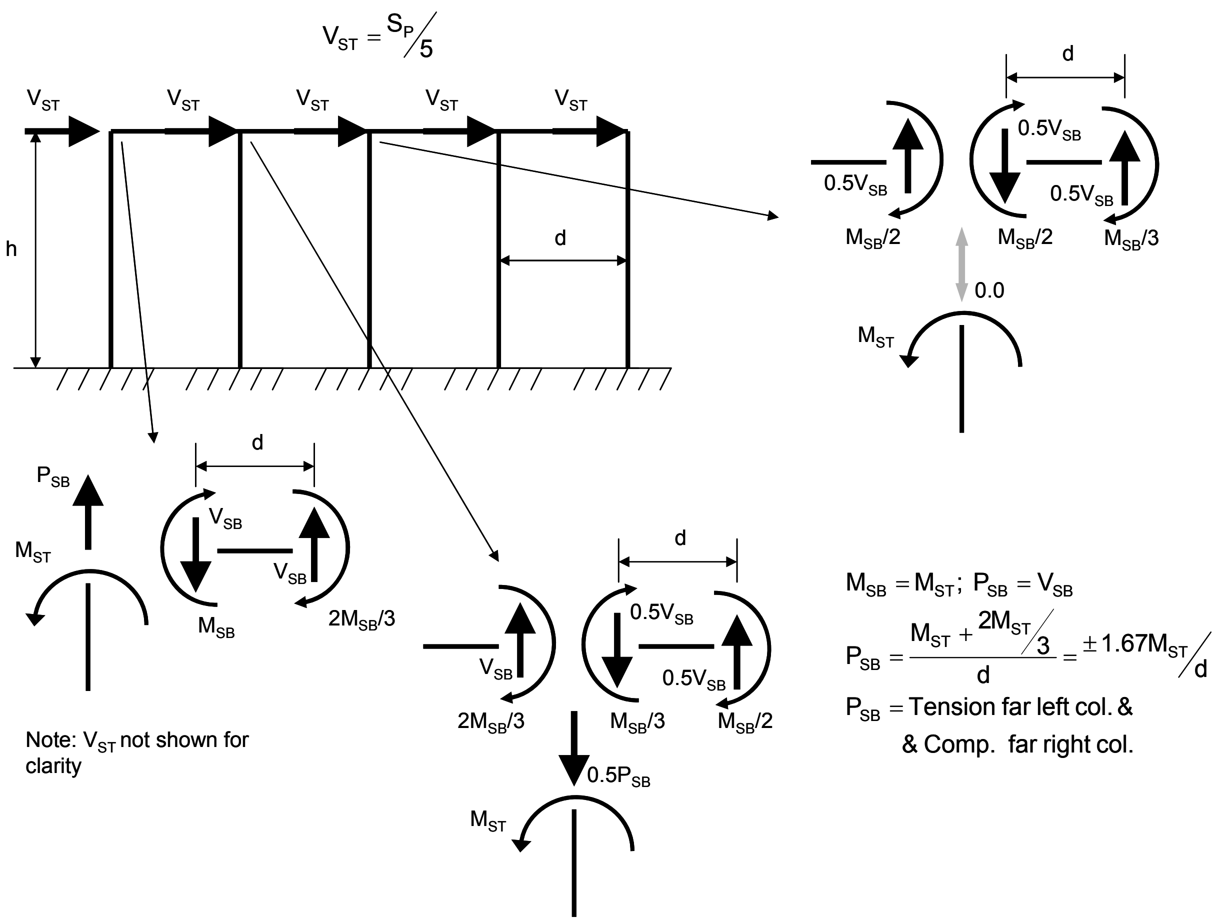


$$M_{SB} = M_{ST};\ P_{SB} = V_{SB}$$

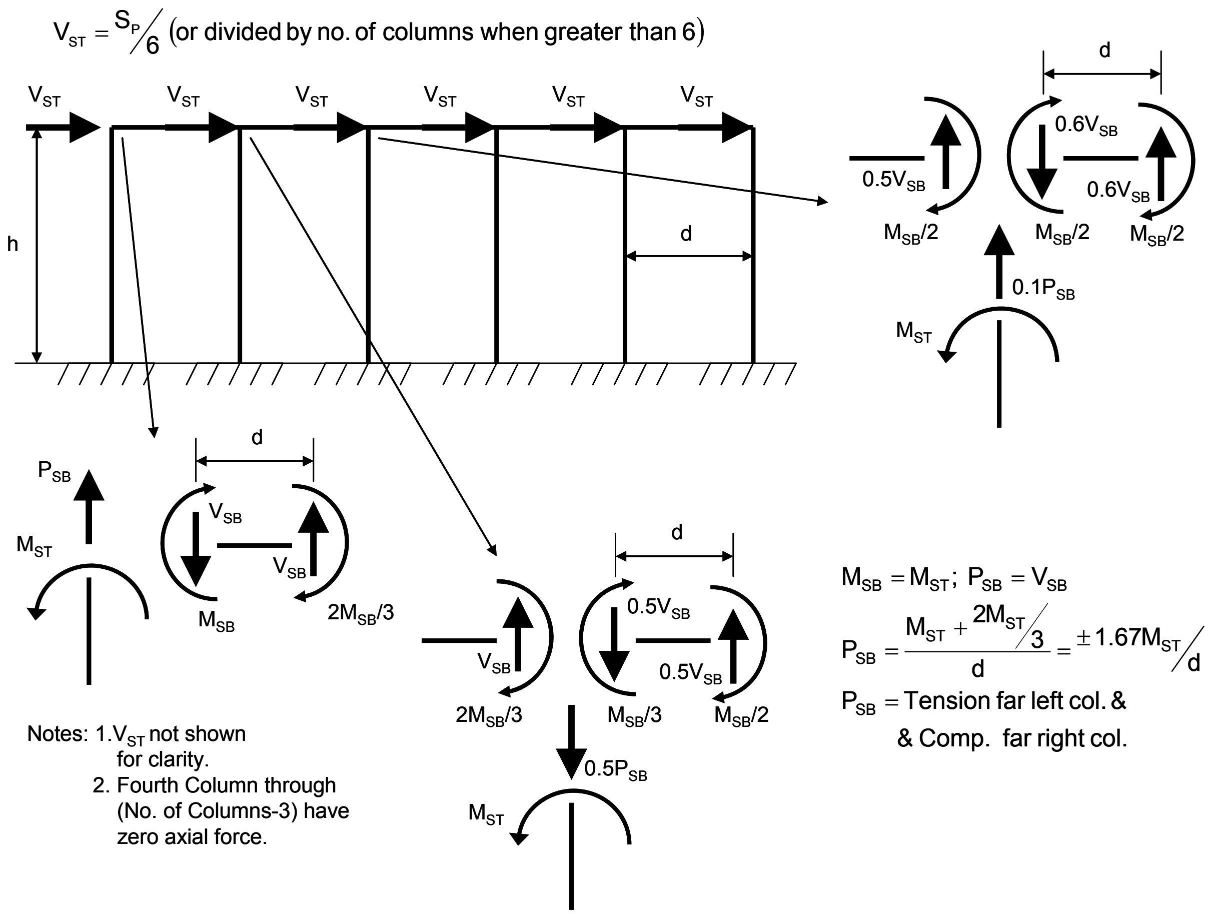
$$P_{SB} = \frac{M_{ST} + 2M_{ST}/3}{d} = \pm 1.67M_{ST}/d$$

P_{SB} = Tension far left col. &
& Comp. far right col.

Case B.3: Frame Action Axial Forces for 4 Column Piers or 4 Piles in a Row



Case B.4: Frame Action Axial Forces for 5 Column Piers or 5 Piles in a Row



Case B.5: Frame Action Axial Forces for 6 or More Column Piers or 6 or More Piles in a Row

Appendix C: Approximate Fixity Depths for H-Piles and Metal Shell Piles in Site Class D Soil, and Axial Force-Moment Interaction Diagrams for Metal Shell Piles

Fixity depths which are generally suitable for typical seismic design for a wide range of situations are given in Tables C.1 and C.2. Table C.1 presents depth-of-fixity values for steel H-piles and Table C.2 presents depths-of-fixity for Metal Shell (MS) piles. Three boundary conditions are given. These are fixed-fixed, fixed-pinned, and the average of the two conditions. For H-piles, values for both the weak and strong axes are provided. The provided moments of intertia for MS piles are "equivalent". The shell and the concrete behave as a reinforced concrete column with the shell acting as the reinforcement. The provided moments of interia are at zero axial force and first yield of the steel shells under pure moment in soil.

For all computed depths-of-fixity, the soil was assumed to have properties at the mid-point of Site Class D as defined in Appendix A of Section 3.15 of the Bridge Manual and in Section 3 of the LRFD Code. However, it should be noted that Site Class D soil was only assumed to be in the "local region" of the pile, or about 2 to 3 times a pile's fixity depth (in terms of deflection), and not necessarily in the entire upper 100 ft. of the soil profile. The upper 100 ft. of the soil profile is typically used to determine the actual Site Class and corresponding Site Coefficients used to calculate the design response spectrum for a structure, but is not necessarily relevant for determination of pile fixity depths.

Class D soil can fall into Soil Profile Types II or III as defined in LFD Div. I-A. The Soil Profile or Site Class Definitions in Appendix A of Section 3.15 of the Bridge Manual and the LRFD Code are much more descriptive and numerically specific than those in the LFD Specifications.

The depths-of-fixity given in Tables C.1 and C.2 may be used for soils with properties ranging from the lower bounds of Site Class C to the upper bounds of Site Class E for typical or "regular" bridges in Illinois (see Section 3.15.3.2 of the Bridge Manual for more information). The upper bounds of Site Class C soils tend to approach the properties of rock and are very stiff. Pile foundations may not be applicable for such soils or piles may be set into rock (for which the depth-of-fixity would be known). The lower bounds of Site Class E tend to approach poor site conditions for which special geotechnical and structural considerations are necessary. Some engineering judgment concerning the use of Tables C.1 and C.2 is required for borderline cases.

The depths-of-fixity given in Tables C.1 and C.2 are primarily intended for seismic design. Engineering judgment is required when using these tables for non-extreme event loadings. Lateral load versus deflection curves (that are non-linear) were generated for each considered case up to structural deformations which were just past initial yielding of the steel. Linear regression was used to fit straight lines to the generated curves. The slope of each fitted line represents a simplified stiffness (and depth-of-fixity) about the axes and for the boundary conditions considered for each pile. The structural stiffness of piles that are at or beyond initial yielding of the steel are “softer” than for non-extreme event loading conditions.

For typical bridge designs, the fixed-fixed case for piles is appropriate for many situations if the guidelines given in Section 3.10 and 3.15 of the Bridge Manual are followed for seismic design. However, cases where fixed-pinned is appropriate are also common. The fixity depths given in Tables C.1 and C.2 are presented in terms of moment at initial yielding of the steel and not deflection. For the fixed-fixed boundary condition case, fixity depths based upon moment and deflection are typically similar. However, for the fixed-pinned case, fixity depths based upon deflection are typically about twice that of the depths based upon moment. When required for global analytical models of bridges, it is more accurate to calculate substructure/foundation stiffnesses with depths-of-fixity based upon deflection. For local analytical models, though, especially when used in conjunction with Appendix B, it is more accurate to calculate design forces with depths-of-fixity based upon moment. See Examples 1 and 4 for applications of these principles in conjunction with Tables C.1 and C.2.

Figures C.1 through C.4 provide nominal axial strength vs. moment strength interaction curves for MS piles. These may be used for structural design of the piles as columns in soil and are not an indicator of geotechnical capacity. MS piles may also have supplemental longitudinal (vertical) and shear/confinement (spiral) reinforcement provided in order to increase structural moment, axial force, and/or shear capacity. Figures C.1 through C.4 are for MS piles without supplemental reinforcement. Note that, in general, the details for reinforcement provided inside of MS piles on the Departmental Base Sheet (F-MS) at abutments are not considered structural for resisting moment or axial force from seismic loadings and should not be considered for confinement or shear. The depth of vertical and spiral reinforcement shown on the Departmental Base Sheet for MS piles is typically insufficient for development of plastic hinges associated with seismic design principles. However, if piles are embedded at least 2 ft. – 0 in. into piers or abutments, the details may be considered as anchorage reinforcement without modification. The individual encasement detail for piles at piers on Base Sheet F-MS should never be considered structural. See Section 3.15.5.5 of the Bridge Manual and Part 14 of Example 4 for additional guidance.

The information on fixity depths given below may be used at the designer's discretion. More complex analyses and refined stiffness determinations are not discouraged by the Department. When more accurate methods are used to determine pile fixity depths, the tables below can be used a guide to gauge whether a more sophisticated analysis is "in the ballpark".

The fixity depths presented in Tables C.1 and C.2 have been established for a variable range of soil conditions and design parameters intended to encompass a large population of structures. Given the variability that is likely to exist between a subject structure and the assumptions used in establishing the fixity depths, it is considered permissible to neglect "group action" factors when using the fixity depths presented in Tables C.1 and C.2.

Pile	Mom. of Inertia Strong Axis (in.^{4})	Fix-Fix Depth of Fixity (ft.)	Fix-Pin Depth of Fixity (ft.)	Average Depth of Fixity (ft.)	Mom. of Inertia Weak Axis (in.^{4})	Fix-Fix Depth of Fixity (ft.)	Fix-Pin Depth of Fixity (ft.)	Average Depth of Fixity (ft.)
HP 14x117	1220	12.5	6.3	9.4	443	9.8	4.8	7.3
HP 14x102	1050	12.0	6.1	9.1	380	9.3	4.6	7.0
HP 14x89	904	11.6	5.8	8.7	326	8.9	4.4	6.7
HP 14x73	729	10.9	5.5	8.2	261	8.4	4.2	6.3
HP 12x84	650	10.9	5.5	8.2	213	8.3	4.1	6.2
HP 12x74	569	10.5	5.3	7.9	186	8.0	4.0	6.0
HP 12x63	472	10.0	5.1	7.6	153	7.7	3.8	5.7
HP 12x53	393	9.6	4.9	7.2	127	7.3	3.6	5.5
HP 10x57	294	9.3	4.7	7.0	101	7.2	3.6	5.4
HP 10x42	210	8.6	4.4	6.5	71.7	6.7	3.3	5.0
HP 8x36	119	8.0	4.0	6.0	40.3	6.2	3.1	4.6

Table C.1: Fixity Depths in Site Class D Soil for H-Piles

Pile	Equiv. Mom. of Inertia Axis (in.^{4})	Fix-Fix Depth of Fixity (ft.)	Fix-Pin Depth of Fixity (ft.)	Average Depth of Fixity (ft.)
MS 14x0.312	420	9.5	4.7	7.1
MS 14x0.25	358.4	9.1	4.5	6.8
MS 12x0.25	214.2	8.3	4.1	6.2
MS 12x0.179	163.2	7.7	3.8	5.8

Table C.2: Fixity Depths in Site Class D Soil for Metal Shell Piles

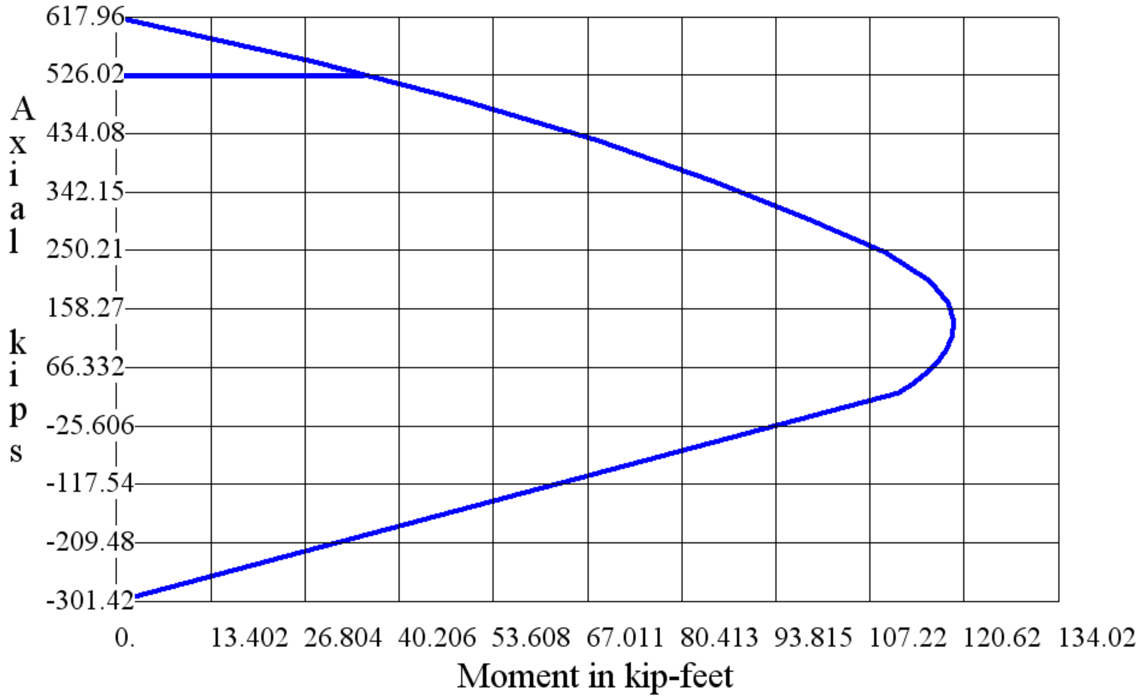


Figure C.1: Nominal Axial Strength vs. Moment Strength Interaction Diagram for 12 x 0.179 in. Metal Shell Piles with F_y = 45 ksi and f'_c = 3.5 ksi

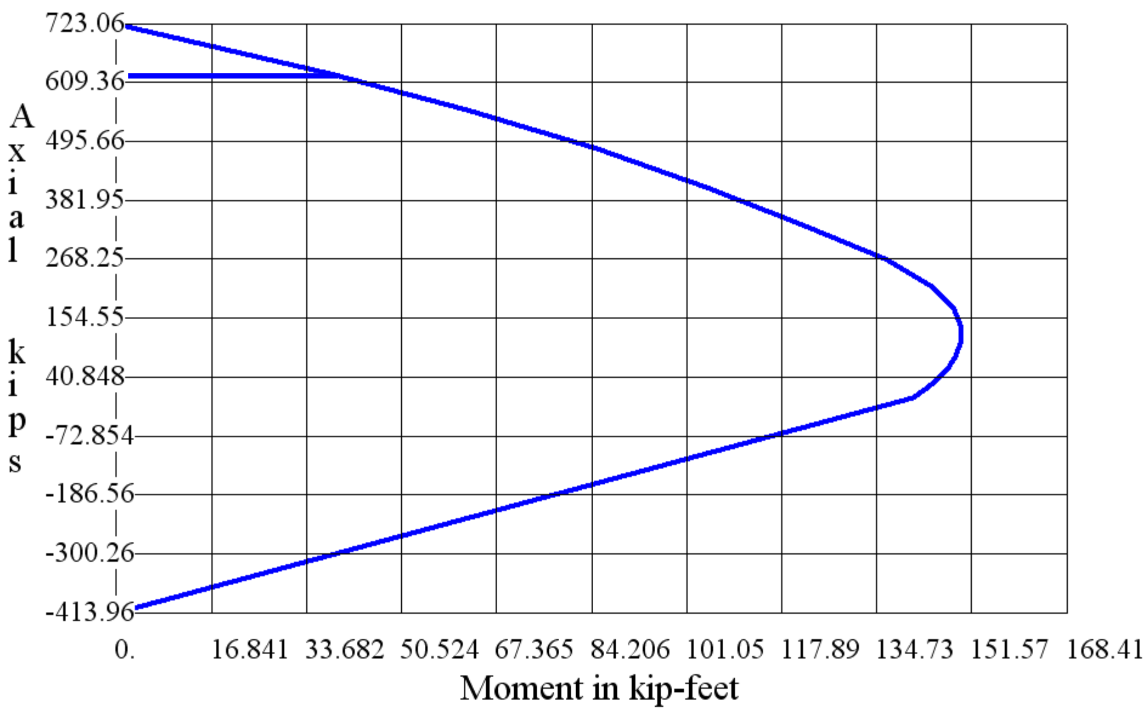


Figure C.2: Nominal Axial Strength vs. Moment Strength Interaction Diagram for 12 x 0.25 in. Metal Shell Piles with F_y = 45 ksi and f'_c = 3.5 ksi

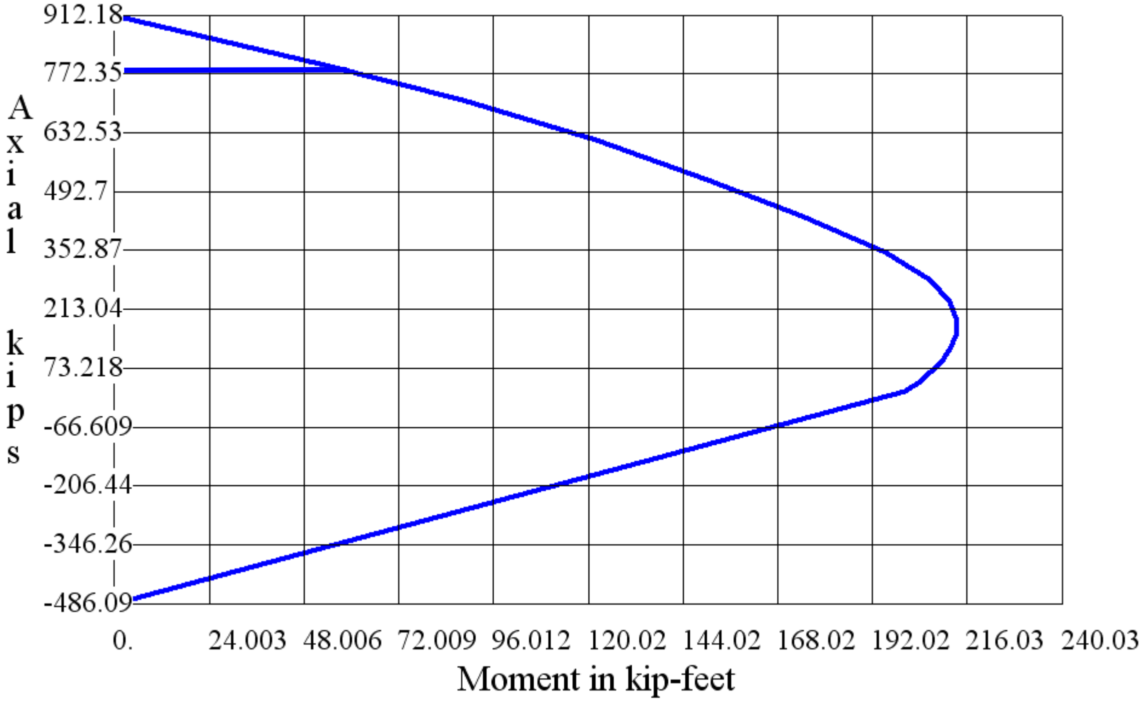


Figure C.3: Nominal Axial Strength vs. Moment Strength Interaction Diagram for 14 x 0.25 in. Metal Shell Piles with F_y = 45 ksi and f'_c = 3.5 ksi

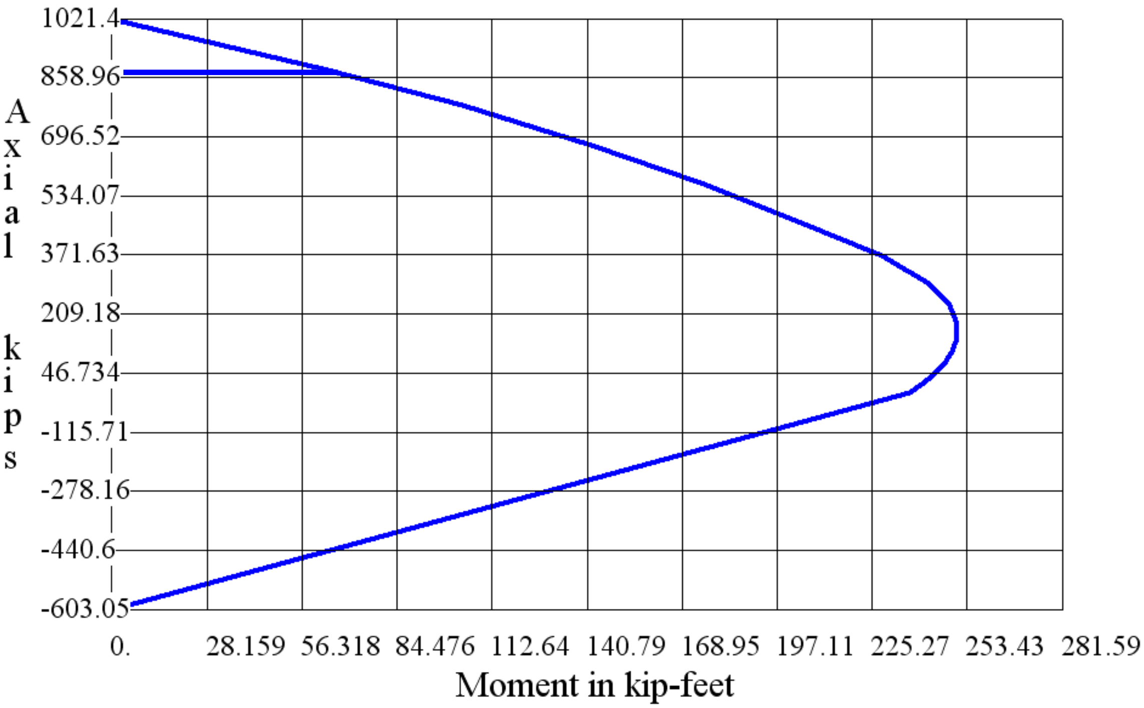


Figure C.4: Nominal Axial Strength vs. Moment Strength Interaction Diagram for 14 x 0.312 in. Metal Shell Piles with F_y = 45 ksi and f'_c = 3.5 ksi