General Requirements

Top slabs and sidewalls of cast in place reinforced concrete culverts are investigated for one-way action.

The standard single box culvert designs in the Culvert Manual were developed assuming a reinforcement ratio of $0.375\rho_{bal}$, and designers should try to meet this goal ratio whenever possible. Top slabs of culverts with fills between 2 feet and 25 feet and sidewalls of culverts with this reinforcement ratio are generally not controlled by shear. Slabs with this reinforcement ratio will be approximately 1½ in. to 2 in. thicker than those allowed if ρ_{max} is used. This additional thickness provides a significant increase in shear capacity. In a minor way, it also reduces the factored shear that must be designed for because it moves the critical section away from support. If designers choose to use a thin slab for projects where the absolute minimum slab thickness is required to satisfy profile grade requirements, a significant increase in the reinforcement will be required.

Article 12.11.1 states the applicable provisions of this section shall be used except as provided otherwise. There are two references to shear in Section 12, 12.5.5 and 12.11.4.1, that modify the shear requirements found in Section 5: Concrete Structures.

Article 12.5.5 and Table 12.5.5-1 Resistance Factors for Buried Structures indicates the resistance factor, φ, for shear in cast in place reinforced concrete box culverts is equal to 0.85.

Article 12.11.4.1 indicates shear shall be investigated according to Article 5.12.7.3. The provisions of Article 5.7 apply except as modified by Article 5.12.7.3. The method of determining the shear resistance of culvert slabs depends on fill height. For culverts with fills ≥ 2 feet, the shear resistance, V_c, of culvert slabs shall be according to the provisions in Article 5.12.7.3. For culverts with fills < 2 feet, the shear resistance of culvert slabs shall be according to the provisions in Article 5.7 and 5.12.8.6. Regardless of fill height, the shear resistance for sidewalls shall be according to the provisions in Articles 5.7 and 5.12.8.6.

Article 5.7 includes the general provisions for shear and Article 5.12.8.6 covers the location of the critical section and makes a statement that one way action is to be designed according to Article 5.7.3.2.

According to Article 5.7.2.1, the factored shear resistance shall be taken as φV_n. According to Article 5.7.3.3, the nominal shear resistance, V_n, is the lesser of:

$$V_n = V_c + V_s + V_p \qquad \text{Eq. 5.7.3.3-1}$$

$$V_n = 0.25 f'_c b_v d_v + V_p \qquad \text{Eq. 5.7.3.3-2}$$

Because shear reinforcement is not used in culvert slabs and the sections are not prestressed, V_s and V_p are both equal to zero. Equation 5.7.3.3-2 does not apply to members without shear reinforcement so no further consideration is needed. Therefore:

$$V_n = V_c$$

The factored shear resistance for all culverts, regardless of fill height, is:

$$\varphi V_n = \varphi V_c = 0.85 V_c$$

While the AASHTO LRFD specifications allow for an increase in shear resistance due to the effect of compressive axial force, the axial force found in culverts with < 25 feet of fill has little effect. This increase may be conservatively ignored for all fill heights.

Critical Section

Article 5.12.8.6.1 states that the critical section for shear is determined according to Article 5.7.3.2, which in turn states that when the reaction introduces compression into the end region of a member, the critical section is located a distance d_v from the internal face of the support.

The provisions for calculating the effective shear depth, d_v, are found in Article 5.7.2.8. The effective shear depth is defined as the distance between the resultants of the compression force and the tension force due to flexure but need not be less than the greater of $0.9d_e$ or $0.72h$.

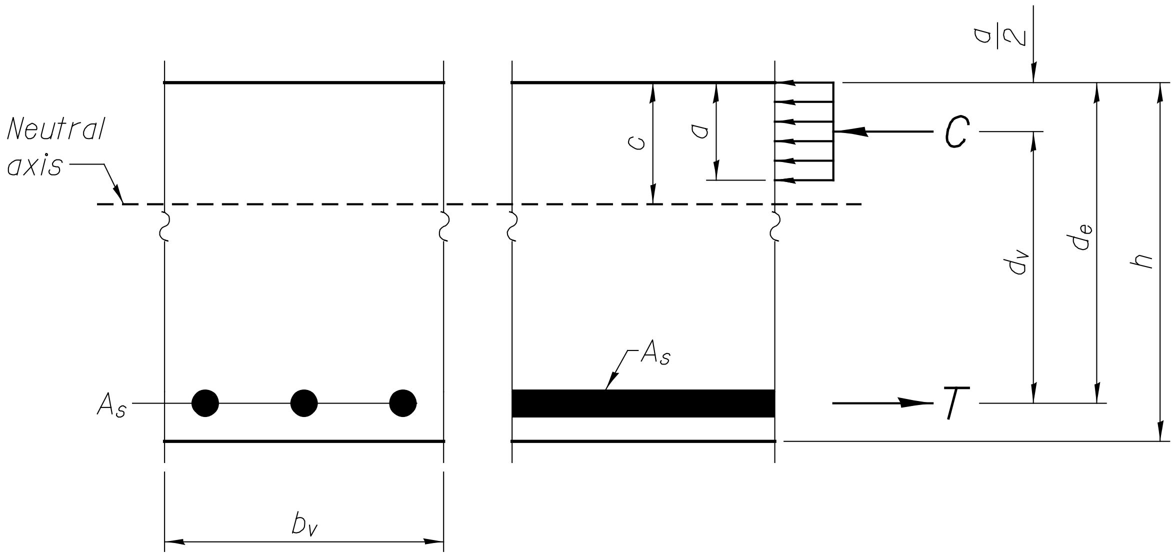

Figure 1 – Effective Shear Depth

For the typical culvert top slab or sidewall with one layer of primary flexural reinforcement:

$$d_v = d_e - \frac{a}{2}$$

where a is the depth of the equivalent compression block calculated according to Article 5.6.2.2 (and defined in Article 5.6.3.2.3) and is equal to $\beta_1 c$. For 3500 psi concrete, β_1 and α_1 are equal to 0.85. The value of c is given by Equation 5.6.3.1.2-4. Because there is no prestressing steel or compression reinforcement, the equation for c becomes:

$$c = \frac{A_s f_y}{0.85 f'_c \beta_1 b}$$

Therefore:

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

d_e is calculated using Equation 5.7.2.8-2. Because this is a non-prestressed member, this equation becomes:

$$d_e = d_s$$

d_s is defined in Articles 5.6.2.1 and 5.6.3.2.2 as the distance from the extreme compression fiber to the centroid of the tensile reinforcement.

For the design of cast in place box culverts with fills < 25 feet, the location of the critical section may conservatively be calculated using d_v equal to 0.72h for the purpose of calculating the factored shear force. Alternatively, for slabs between 12 in. and 21 in. thick, d_v can be approximated as 0.75h. For slabs ≥ 21 in. thick, d_v can be approximated as 0.80h. The use of these values will provide a conservative value for the factored shear force.

For the design of cast in place box culverts with fills > 25 feet, the location of the critical section may initially be calculated using d_v equal to 0.80h. Through an iterative process, the slab thickness should be refined such that the factored force is calculated at the correct location.

The resistance of the section should be based on the actual value of d_v or d_e, as appropriate.

Requirements For Fills ≥ 2 Feet

For fills ≥ 2 feet, the specific provisions of Article 5.12.7.3 are used to calculate V_c.

The AASHTO LRFD specifications place the critical section at a distance d_v from the face of the support.

At fills > 25 feet, shear will most likely control the design and the designer should take care to properly determine the factored shear forces at the actual critical section rather than at a conservative estimate of the critical section.

$$V_c = \left(0.0676\sqrt{f'_c} + 4.6\frac{A_s}{bd_e}\frac{V_u d_e}{M_u}\right) bd_e \leq 0.126\sqrt{f'_c}\, bd_e$$

Eq. 5.12.7.3-1 &5.12.7.3-2

And

$$\frac{V_u d_e}{M_u} \leq 1.0$$

But for single-cell boxes with simply supported walls, need not be less than:

$$0.0791\sqrt{f'_c}\, bd_e$$

For 3500 psi cast in place concrete and a unit width of 12 in., the equation for the lower limit for V_c is equal to:

$$V_c = 0.0791\sqrt{3.5\text{ ksi}}\,(12\text{ in.})d_e = 1.78d_e$$

Therefore, the lower limit for φV_n is:

$$\varphi V_n = (0.85)1.78d_e = 1.51d_e$$

The lower limit is adequate to satisfy all single box culvert top slabs up to 20 feet of fill if the slab thickness is proportioned to satisfy flexural requirements with the primary flexural reinforcement limited to $0.375\rho_{bal}$ and the bar size of the primary flexural reinforcement is limited such that the bar may be fully developed. If the designer chooses to use Equations 5.12.7.3-1 and 5.12.7.3-2 instead of the lower limit, a reasonable initial assumption for the value of $\frac{V_u d_e}{M_u}$ is 0.75. This value should be verified during the final design.

The upper limit of Equations 5.12.7.3-1 and 5.12.7.3-2 will not control for any concrete slabs typically found in culvert construction for concrete strength of 3500 psi or greater as long as the flexural reinforcement is no greater than $\rho_{max} = 0.75\rho_{bal}$. The definitions of ρ_{max} and ρ_{bal} are found in the AASHTO Standard Specification for Highway Bridges.

While not explicitly stated in Article 5.12.7.3, designers must account for the development (or lack thereof) of the primary reinforcement similar to the procedure when using Equation 5.7.3.4.2-4. The bar size limitations shown elsewhere in this design guide were developed to ensure full (or near full) development of the bar. The development of the reinforcement should always be checked and the area of primary reinforcement used in Equations 5.12.7.3-1 and 5.12.7.3-2 should be adjusted to account for any lack of development.

The following table provides the factored shear resistances per one foot width for top slabs of single box culverts with fills ≥ 2 feet of the given thickness and reinforcement (approximately equal to $0.375\rho_{bal}$) assuming the reinforcement is fully developed and $\frac{V_u d_e}{M_u} = 0.75$.

Top Slab Thickness (in.)	Bar	Spacing (in.)	Area (in.^{2}/ft.)	d_e (in.)	φV_n (kips)	
					$0.0791\sqrt{f'_c}\, bd_e$	Eq. 5.12.7.3-1 $\left(\frac{V_u d_e}{M_u}=0.75\right)$
6	4	5.5	0.44	3.75	5.66	6.12
6.5	5	7.5	0.50	4.19	6.32	6.86
7	5	7	0.53	4.69	7.08	7.61
7.5	5	6	0.62	5.19	7.83	8.51
8	6	7.5	0.70	5.63	8.49	9.32
8.5	6	7.5	0.70	6.13	9.25	9.97
9	6	7	0.75	6.63	10.00	10.76
9.5	6	6.5	0.81	7.13	10.75	11.57
10	6	6	0.88	7.63	11.51	12.42
10.5	7	7	1.03	8.06	12.17	13.42
11	7	7	1.03	8.56	12.92	14.06
11.5	7	7	1.03	9.06	13.68	14.71
12	7	6.5	1.11	9.56	14.43	15.58
12.5	7	6	1.20	10.06	15.19	16.50
13	7	6	1.20	10.56	15.94	17.14
13.5	7	5.5	1.31	11.06	16.70	18.11
14	7	5.5	1.31	11.56	17.45	18.75
14.5	7	5	1.44	12.06	18.21	19.78
15	8	6.5	1.46	12.50	18.87	20.40
15.5	8	6	1.58	13.00	19.62	21.40
16	8	6	1.58	13.50	20.38	22.05
17	8	5.5	1.72	14.50	21.89	23.76
18	9	6.5	1.85	15.44	23.30	25.33
19	9	6.5	1.85	16.44	24.81	26.62
20	9	6	2.00	17.44	26.32	28.36
21	10	7	2.18	18.37	27.72	30.07
22	10	7	2.18	19.37	29.23	31.36

Requirements For Fills < 2 Feet

For fills < 2 feet, the provisions of Article 5.7.3.3 are used to calculate V_c. While Article 5.7.3.4.1 provides a simplified procedure for the calculation of β for slabs thinner than 16 in., designers should always calculate β according to the general procedure in Article 5.7.3.4.2. For the application of Article 5.7.3.4.2 see Worked Example 2. Top slabs of single box culverts with fills < 2 feet will be governed by shear if the simplified procedure is used, resulting in thicker than required slabs.

$$V_c = 0.0316\beta\sqrt{f'_c}b_v d_v \qquad \text{Eq. 5.7.3.3-3}$$

Therefore:

$$\varphi V_n = 0.85V_c = (0.85)0.0316\beta\sqrt{f'_c}b_v d_v$$

For 3500 psi cast in place concrete and a unit width of 12 in., the equation for φV_n is:

$$\varphi V_n = (0.85)0.0316\beta\sqrt{3.5\text{ ksi}}(12\text{ in.})d_v = 0.603\beta d_v$$

A reasonable assumption for β for a single box culvert top slab is 2.7 and for d_v is 0.72h. By combining these assumptions, a conservative value for the factored resistance of the slab is:

$$\varphi V_n = 0.603(2.7)0.72h = 1.17h$$

Designers should always calculate β for all culverts with fills < 2 feet.

The second point toward the end of Article 5.7.3.4.2 requires designers to account for the development (or lack thereof) of the primary reinforcement when using Equation 5.7.3.4.2-4. The bar size limitations shown elsewhere in this design guide were developed to ensure full (or near full) development of the bar. The development of the reinforcement should always be checked and the area of primary reinforcement used in Equation 5.7.3.4.2-4 should be adjusted to account for any lack of development.

Requirements For Sidewalls

Sidewalls are designed using the same provisions as used for culverts with fills < 2 feet.

$$\varphi V_n = \varphi V_c = (\varphi)0.0316\beta\sqrt{f'_c}b_v d_v$$

For 3500 psi cast in place concrete and a unit width of 12 in., the equation for φV_n is:

$$\varphi V_n = (0.85)0.0316\beta\sqrt{3.5\text{ ksi}}(12\text{ in.})d_v = 0.603\beta d_v$$

The value of β varies with fill heights. For fills < 25 feet, β may conservatively be assumed equal to 2.0 for sidewalls thinner than 16 in. and have no impact on the design. If a more accurate approximation is desired, for fills ≤ 12 feet, β = 3.0 and for fills between 12 and 25 feet, β = 2.7. These initial values should be verified by further calculation. As in the case of single box culverts top slabs with fills < 2 feet, the development of the reinforcement should be checked and the area of primary reinforcement used in Equation 5.7.3.4.2-4 should be adjusted to account for any lack of development. Designers should verify the development of the reinforcement. If the reinforcement is found to not be fully developed the general procedure of Article 5.7.3.4.2 should be used.

Conservatively:

$$\varphi V_n = 0.603(2.0)0.72h = 0.87h$$

For fills ≤ 12 feet:

$$\varphi V_n = 0.603(3.0)0.72h = 1.30h$$

For 12<fills ≤ 25 feet:

$$\varphi V_n = 0.603(2.7)0.72h = 1.17h$$

The following table provides the factored shear resistances per one foot width for single box culvert top slabs with fills < 2 feet and sidewalls of the given thickness and reinforcement for the values of β previously discussed. The lower limit of the shear resistance shown in the table was determined by applying a factored shear and moment to the slab, equal to the shear and moment resistance, and calculating the true value of β for that loading. This loading is the worst case for the section and these values of β may be used in lieu of the value of 2.0 given in the AASHTO LRFD specifications if the reinforcement is equal to or greater than $0.375\rho_{bal}$. An increase in the area of primary flexural reinforcement or a reduction in the concurrent moment will cause a significant increase in the value of β and a corresponding increase in shear resistance.

Thickness (in.)	Bar	Spacing (in.)	Area (in.^{2}/ft.)	d_v (in.)	Lower Limit		φV_n (kips) Beta, β		
					Beta, β	φV_n (kips)	2.0	2.7	3.0
6	4	5.5	0.44	4.32	2.29	5.97	5.21	7.03	7.81
6.5	5	7.50	0.50	4.68	2.26	6.38	5.64	7.62	8.47
7	5	7.00	0.53	5.04	2.21	6.72	6.08	8.21	9.12
7.5	5	6.00	0.62	5.40	2.18	7.10	6.51	8.79	9.77
8	6	7.50	0.70	5.76	2.16	7.50	6.95	9.38	10.42
8.5	6	7.50	0.70	6.12	2.12	7.82	7.38	9.96	11.07
9	6	7.00	0.75	6.48	2.09	8.17	7.81	10.55	11.72
9.5	6	6.50	0.81	6.84	2.07	8.54	8.25	11.14	12.37
10	6	6.00	0.88	7.20	2.06	8.94	8.68	11.72	13.02
10.5	7	7.00	1.03	7.56	2.06	9.39	9.12	12.31	13.68
11	7	7.00	1.03	7.92	2.04	9.74	9.55	12.89	14.33
11.5	7	7.00	1.03	8.28	2.01	10.04	9.99	13.48	14.98
12	7	6.50	1.11	8.64	2.00	10.42	10.42	14.07	15.63
12.5	7	6.00	1.20	9.06	2.00	10.92	10.92	14.74	16.38
13	7	6.00	1.20	9.55	2.00	11.52	11.52	15.56	17.28
13.5	7	5.50	1.31	9.96	2.00	12.01	12.01	16.22	18.02
14	7	5.50	1.31	10.46	1.97	12.43	12.62	17.03	18.93
14.5	7	5.00	1.44	10.86	1.96	12.83	13.09	17.68	19.64
15	8	6.50	1.46	11.27	1.94	13.19	13.60	18.36	20.40
15.5	8	6.00	1.58	11.70	1.92	13.55	14.11	19.05	21.17
16*	8	6.00	1.58	12.17	1.90	13.95	14.68	19.82	22.02
17*	8	5.50	1.72	13.05	1.86	14.64	15.74	21.25	23.61
18*	9	6.50	1.85	13.89	1.83	15.33	16.75	22.62	25.13
19*	9	6.50	1.85	14.88	1.79	16.07	17.95	24.23	26.93
20*	9	6.00	2.00	15.76	1.75	16.63	19.00	25.65	28.50
21*	10	7.00	2.18	16.54	1.73	17.25	19.94	26.92	29.91
22*	10	7.00	2.18	17.54	1.69	17.87	21.15	28.55	31.72

*The simplified procedure in Article 5.7.3.4.1 where $\beta = 2.0$ is only directly applicable for members less than 16 inches in thickness. The values for $\beta = 2.0$ may be used for preliminary design but the “actual” value of β should be computed using the general procedure in Article 5.7.3.4.2 during the final design. The simplified procedure is inherently conservative for slabs less than 16 in. thick but may be conservative for thicker slabs.

Worked Example 1: Culvert Top Slab with Fill ≥2 Feet

Determine the shear capacity of the top slab of a cast in place single 10' x 4' box culvert with 7 feet of fill. The top slab is 11½ in. thick with #7 bars at 7 in. centers and the bottom slab is 12½ in. thick. The sidewalls are 6 in. thick. The bars are not epoxy coated and the clear cover to the reinforcement is 2 in. For the purpose of determining the factored shear force, the critical section is assumed to be located at a distance equal to 0.72h from the inside face of the sidewall which is equal to 8.28 in. The factored shear force at the critical section is 9.2 kips and the concurrent moment is 9.4 k-ft. Neglect the effect of axial force. f'_c = 3500 psi, f_y = 60000 psi.

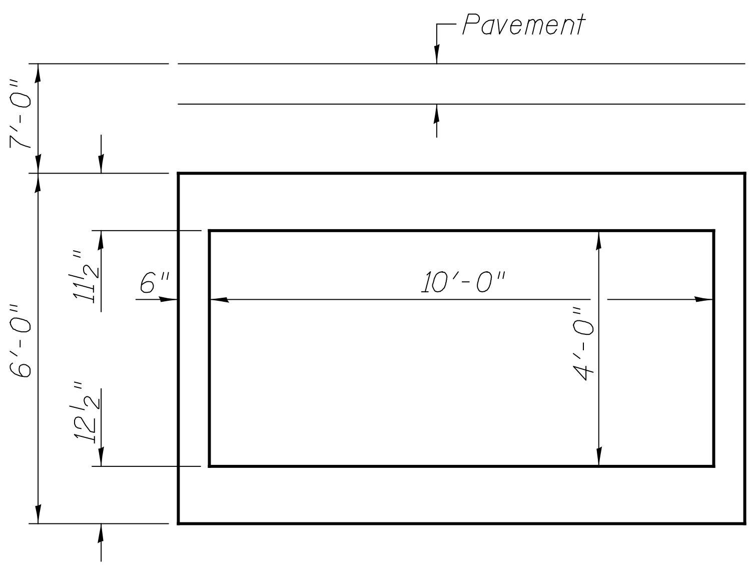

Figure 2 – Worked Example 1 – Culvert Cross Section

Using the provisions of Article 5.12.7.3:

$$V_c = \left(0.0676\sqrt{f'_c}+4.6\frac{A_s}{bd_e}\frac{V_u d_e}{M_u}\right)bd_e \le 0.126\sqrt{f'_c}\,bd_e \qquad \text{Eq. 5.12.7.3-1}$$

And

$$\frac{V_u d_e}{M_u} \le 1.0$$

But for single cell boxes with simply supported walls, V_u need not be less than:

$$0.0791\sqrt{f_c'}\,bd_e$$

Calculate the distance from the centroid of the tension steel to the extreme compression fiber, d_e.

$$d = t_{topslab} - cover - \frac{d_b}{2} = 11.5\text{ in.} - 2\text{ in.} - \frac{0.875\text{ in.}}{2} = 9.06\text{ in.}$$

$$d_e = d = 9.06\text{ in.}$$

Calculate the effective shear depth, d_v.

For flexure, the area of steel provided is:

$$A_s = (0.60\text{ in.}^2)\frac{12\text{ in.}}{7\text{ in.}} = 1.03\text{ in.}^2$$

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

$$a = \frac{(1.03\text{ in.}^2)(60\text{ ksi})}{0.85(3.5\text{ ksi})(12\text{ in.})} = 1.73\text{ in.}$$

$$d_v = d - \frac{a}{2} = 9.06\text{ in.} - \frac{1.73\text{ in.}}{2} = 8.20\text{ in.}$$

but need not be less than the greater of:

$$0.9d_e = 0.9(9.06\text{ in.}) = 8.15\text{ in.}$$

or

$$0.72h = 0.72(11.5\text{ in.}) = 8.28\text{ in.}$$

$$d_v = 8.28\text{ in.}$$

Verify the development of the #7 bar chosen to satisfy flexural requirements. Modify A_s proportional to the lack of development if the bar is found to not be fully developed at the critical section. While this is not explicitly written in Article 5.12.7.3, the concern of lack of development is the same as that noted in Article 5.7.3.4.2.

Calculate the development length of a #7 bar with a standard 180 degree hook per Article 5.10.8.2.4.

$$l_{hb} = \frac{38.0d_b}{60}\left(\frac{f_y}{\lambda\sqrt{f'_c}}\right) = \frac{38.0(0.875\text{ in.})}{60}\left(\frac{60\text{ ksi}}{1.0\sqrt{3.5\text{ ksi}}}\right) = 17.77\text{ in.}$$

Apply modification factors per Article 5.10.8.2.4b.

$$l_{dh} = l_{hb}\left(\frac{\lambda_{rc}\lambda_{cw}\lambda_{er}}{\lambda}\right)$$

λ_{rc} = modification factor for reinforcement confinement, taken as 0.8 for #11 bars and smaller with spacing exceeding $3d_b$.

λ_{cw} = modification factor for epoxy-coated reinforcement = 1.2 for hooked reinforcement

λ_{er} = modification factor for sections with more reinforcement than required for strength. Assume λ_{er} = 1.0 for shear calculations.

For this example:

$$l_{dh} = 1.2(0.8)(17.77\text{ in.}) = 17.06\text{ in.}$$

Determine if the #7 bar is developed at the critical section. Assume a 2 in. cover from the edge of the slab to the outside bend of the hook.

$$D_{cs} = t_{sidewall} + d_v - 2\text{ in.} = 6\text{ in.} + 8.28\text{ in.} - 2\text{ in.} = 12.28\text{ in.} < 17.06\text{ in.}$$

The hooked #7 bar is not fully developed at the critical section, therefore A_s is not fully effective.

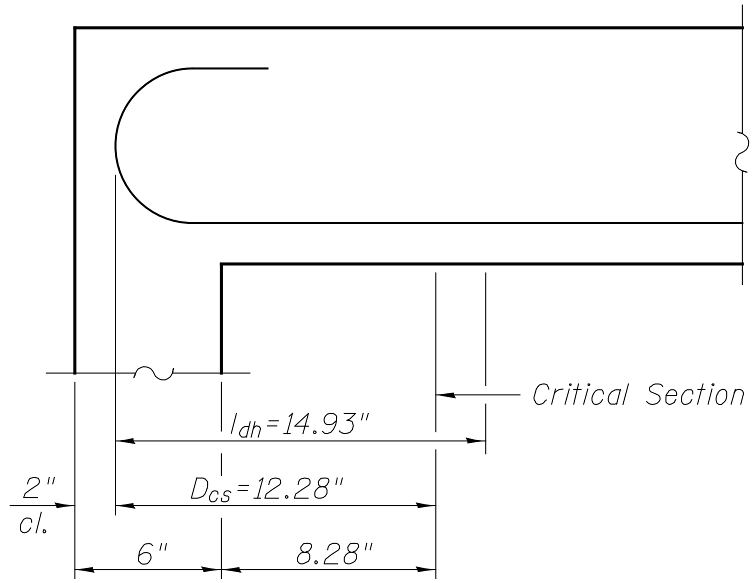

Figure 3 – Worked Example 1 – Location of Bar Development for Shear

Reduce the area of steel used in Equation 5.12.7.3-1 to account for the lack of development.

$$A_s=(1.03\text{ in.}^2)\frac{12.28\text{ in.}}{17.06\text{ in.}}=0.74\text{ in.}^2$$

Calculate the nominal shear resistance.

$$\frac{V_u d_e}{M_u}=\frac{(9.2\text{ k})(9.06\text{ in.})}{(9.4\text{ k-ft.})(12)}=0.739<1\quad\text{OK}$$

$$V_c=\left(0.0676\sqrt{3.5\text{ ksi}}+4.6\frac{0.74\text{ in.}^2}{(12\text{ in.})(9.06\text{ in.})}0.739\right)(12\text{ in.})(9.06\text{ in.})=16.3\text{ k}$$

$$0.0791\sqrt{f'_c}\,bd_e=0.0791\sqrt{3.5\text{ ksi}}\,(12\text{ in.})(9.06\text{ in.})=16.1\text{ k}<16.3\text{ k}$$

$$0.126\sqrt{f'_c}\,bd_e=0.126\sqrt{3.5\text{ ksi}}\,(12\text{ in.})(9.06\text{ in.})=25.6\text{ k}>16.3\text{ k}$$

$$V_n=V_c=16.1\text{ k}$$

Calculate the factored shear resistance, φV_n.

φ=0.85 per Table 12.5.5-1

$$\varphi V_n = 0.85(16.1\text{ k}) = 13.7\text{ k} > 9.2\text{ k} \quad \text{OK}$$

The 11½ in. thick top slab is adequate for shear.

Worked Example 2: Culvert Top Slab with Fill < 2 Feet

Determine the shear capacity of the top slab of a cast in place single 9 ft. x 8 ft. box culvert with 1.5 feet of fill. The top slab is 10½ in. thick with #7 at 7 in. centers and the bottom slab is 11½ in. thick. The sidewalls are 8 in. thick. The bars are epoxy coated and the clear cover to the reinforcement is 2 in. For the purpose of determining the factored shear force, the critical section is assumed to be located at a distance equal to 0.72h from the inside face of the sidewall which is equal to 7.56 in. The factored shear force at the critical section is 10.4 kips and the concurrent moment is 10.3 k-ft. Neglect the effect of axial force.

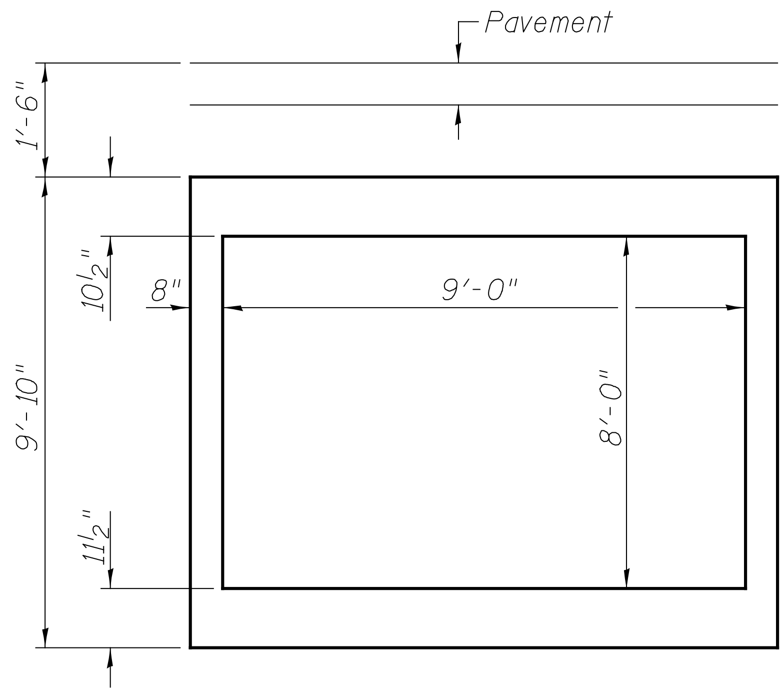

Figure 4 – Worked Example 2 – Culvert Cross Section

Calculate the nominal shear resistance according to Article 5.7.3.3.

$$V_c = 0.0316\beta\sqrt{f_c'}b_v d_v$$

Calculate the distance from the centroid of the tension steel to the extreme compression fiber, d_e.

$$d = t_{topslab} - \text{cover} - \frac{d_b}{2} = 10.5\text{ in.} - 2\text{ in.} - \frac{0.875\text{ in.}}{2} = 8.06\text{ in.}$$

$$d_e = d = 8.06\text{ in.}$$

Calculate the effective shear depth, d_v.

For flexure, the area of steel provided is:

$$A_s = (0.60\text{ in.}^2)\frac{12\text{ in.}}{7\text{ in.}} = 1.03\text{ in.}^2$$

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

$$a = \frac{(1.03\text{ in.}^2)(60\text{ ksi})}{0.85(3.5\text{ ksi})(12\text{ in.})} = 1.73\text{ in.}$$

$$d_v = d - \frac{a}{2} = 8.06\text{ in.} - \frac{1.73\text{ in.}}{2} = 7.20\text{ in.}$$

but need not be less than the greater of:

$$0.9d_e = 0.9(8.06\text{ in.}) = 7.25\text{ in.}$$

or

$$0.72h = 0.72(10.5\text{ in.}) = 7.56\text{ in.}$$

$$d_v = 7.56\text{ in.}$$

Verify the development of the #7 bar chosen to satisfy flexural requirements. Modify A_s proportional to the lack of development if the bar is found to not be fully developed at the critical section.

Calculate the development length of a #7 bar with a standard 180 degree hook per Article 5.11.2.4.

$$l_{dh} = 17.06\text{ in. (see previous example)}$$

Determine if the #7 bar is developed at the critical section. Assume a 2 in. cover from the edge of the slab to the outside bend of the hook.

$$D_{cs} = t_{sidewall} + d_v - 2\text{ in.} = 8\text{ in.} + 7.56\text{ in.} - 2\text{ in.} = 13.56\text{ in.} < 17.06\text{ in.}$$

The hooked #7 bar is not fully developed at the critical section therefore A_s is not fully effective.

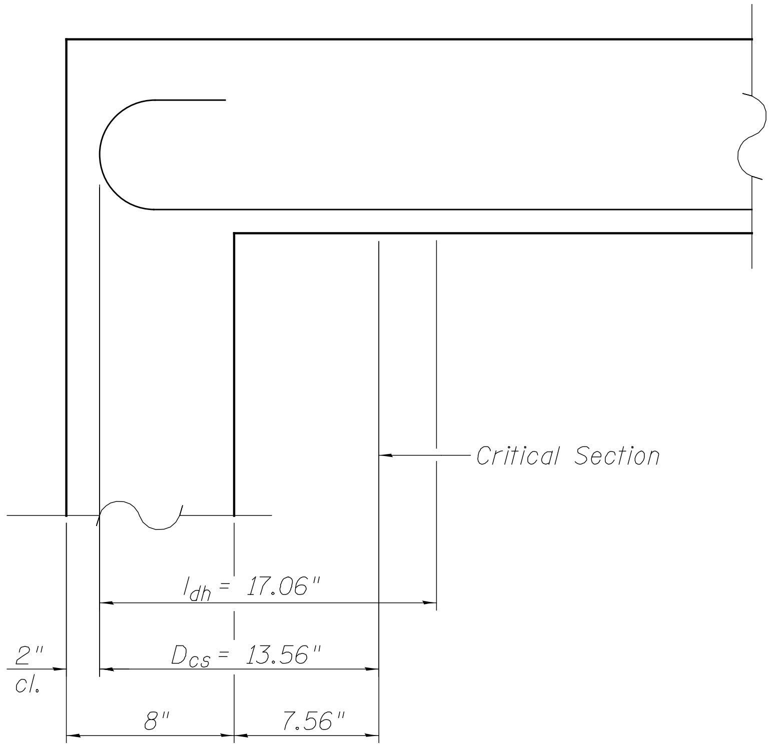

Figure 5 – Worked Example 2 – Location of Bar Development for Shear

Reduce the area of steel used in Equation 5.12.7.3-1 or 5.12.7.3-2 to account for the lack of development

$$A_s = (1.03\text{ in.}^2)\frac{13.56\text{ in.}}{17.06\text{ in.}} = 0.82\text{ in.}^2$$

As the slab thickness is less than 16 in., the simplified procedure ($\beta = 2.0$) in Article 5.7.3.4.1 is applicable. However, as shown below, use of $\beta = 2.0$ can be very conservative. The below example will show both the simplified procedure and the general procedure to show a comparison.

If $\beta = 2.0$ is assumed,

$$V_n = V_c = 0.0316(2.0)\sqrt{3.5\text{ ksi}}(12\text{ in.})(7.56\text{ in.}) = 10.7\text{ k}$$

Calculate the factored shear resistance, φV_n.

$$\varphi = 0.85 \text{ per Table 12.5.5-1}$$

$$\varphi V_n = 0.85(10.7\text{ k}) = 9.1\text{ k} < 10.4\text{ k} \quad \text{NO GOOD}$$

Use the general procedure of Article 5.7.3.4.2 to calculate β.

$$\beta = \frac{4.8}{(1+750\varepsilon_s)}\frac{51}{(39+s_{xe})}$$

Determine the crack spacing parameter, s_{xe} and strain in the bar, ε_s.

$$s_{xe} = s_x \frac{1.38}{a_g+0.63} \geq 12\text{ in.}$$

$$\varepsilon_s = \frac{\left(\frac{|M_u|}{d_v}+|V_u|\right)}{E_sA_s}$$

where:

$$|M_u| \geq |V_u|d_v$$

Calculate the strain in the bar.

$$|V_u|d_v = (10.4\text{ k})(7.56\text{ in.}) = 78.6\text{ k-in.}$$

$$|M_u| = (10.3\text{ k-ft.})(12\text{ in./ft.}) = 124\text{ k-in.} \quad \text{OK}$$

$$\varepsilon_s = \frac{\left(\frac{|124\text{ k-in.}|}{7.56\text{ in.}}+|10.4\text{ k}|\right)}{(29{,}000\text{ ksi})(0.82\text{ in.}^2)} = 0.0011$$

The aggregates used in Class SI concrete are CA 7, CA 11, CA 13, CA 14 and CA 16. The maximum aggregate size ranges from 1.5 in. for CA 7 to 0.5 in. for CA 16.

$$s_x = d_v = 7.56\text{ in.}$$

$$s_{xe} = (7.56 \text{ in.})\frac{1.38}{1.5 \text{ in.} + 0.63} = 4.89 \text{ in.} < 12 \text{ in.}$$

$$s_{xe} = 12 \text{ in.}$$

$$\beta = \frac{4.8}{(1 + 750(0.0011))}\frac{51}{(39 + 12 \text{ in.})} = 2.63$$

$$V_c = 0.0316(2.63)\sqrt{3.5 \text{ ksi}}(12 \text{ in.})(7.56 \text{ in.}) = 14.1 \text{ k}$$

The nominal shear resistance is:

$$V_n = V_c = 14.1 \text{ k}$$

Calculate the factored shear resistance, φV_n.

$$\varphi = 0.85 \text{ per Table 12.5.5-1}$$

$$\varphi V_n = 0.85(14.1 \text{ k}) = 12.0 \text{ k} > 10.4 \text{ k} \quad \text{OK}$$

The 10½ in. thick top slab is adequate for shear. Note the significant increase in shear capacity when the general procedure is used.